DEUTSCHE
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Discussion Paper

Deutsche Bundesbank
No 41/2019

Nowcasting GDP with a large factor model space

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Internet http://www.bundesbank.de

ISBN 978–3–95729–640–5 (Printversion)
ISBN 978–3–95729–641–2 (Internetversion)

Non-technical summary

Research Question

Central banks require a reliable assessment of the past, current and future state of the economy to conduct monetary policy, such as setting key interest rates. Although the recent nowcasting literature has proposed models that can efficiently deal with the properties of the real-time data flow, such as mixed data frequencies and publication lags, these models usually do not allow for time-variation in parameters. We hence ask the question whether we can design a model that can deal with both, the issues implied by the real-time data flow and structural instabilities. If so, can this model add to the nowcasting toolbox by providing more precise forecasts or does the added complexity disrupt any potential improvements?

Contribution

We introduce a novel time-varying parameter mixed-frequency dynamic factor model that can efficiently deal with the issues implied by the real-time data flow as well as parameter change. Moreover, we propose an algorithm that is optimized for fast estimation. This feature not only allows the forecaster to combine forecasts from various model specifications, but also provides additional transparency in the black-box environment of factor models. To sum up, we combine the merits of both, the recent nowcasting literature as well as the literature on time-varying parameters and model averaging.

Results

In an empirical forecasting exercise of the growth rate of the real gross domestic product in Germany, we find that our model can improve upon the forecast performance of competing models. These forecast performance gains arise mostly during periods of turmoil such as the recent financial crisis.

Nichttechnische Zusammenfassung

Fragestellung

Zentralbanken benötigen eine verlässliche Einschätzung der vergangenen, gegenwärtigen und zukünftigen wirtschaftlichen Lage bei geldpolitischen Entscheidungen, wie z.B. beim Festlegen der Leitzinsen. Obwohl die Nowcasting Literatur Modelle hervorgebracht hat, die auf effiziente Weise mit den Eigenschaften des sogenanten „real-time data flow", wie gemischte Frequenzen und Veröffentlichungsverzögerungen, umgehen können, werden Probleme wie Zeitvariation in den Parametern üblicherweise vernachlässigt. Die Frage dieser Forschungsarbeit ist deshalb, ob sich ein Prognosemodell entwicklen lässt, das sowohl die Eigenschaften des sogenannten „real-time data flow", als auch die Zeitvariation der Parameter berücksichtigt. Falls ja, kann dieses Modell präzisere Prognosen liefern oder führt die erhöhte Komplexität zu einem Verlust potenzieller Vorteile?

Beitrag

Wir stellen ein neues dynamisches Faktormodell vor, das sowohl gemischte Frequenzen und Veröffentlichungsverzögerungen, als auch Zeitvariation der Parameter berücksichtigt. Zusätzlich entwickeln wir einen Algorithmus, der eine schnelle Schätzung des Modells erlaubt. Diese Eigenschaft ermöglicht nicht nur die Schätzung unterschiedlicher Modellspezifikationen und die Kombination der daraus resultierenden Prognosen, sondern schafft auch erhöhte Transparenz in Bezug auf die sonst schwer interpretierbaren Faktormodelle. Zusammengefasst kombinieren wir die Vorteile der jungen Nowcasting Literatur und der Literatur, die sich mit zeitvariierenden Parametern und der Kombination von Prognosen beschäftigt.

Ergebnisse

In einer empirischen Anwendung auf das Wachstum des realen Bruttoinlandsprodukts in Deutschland stellen wir fest, dass unser Modell im Vergleich zu konkurierenden Modellen eine verbesserte Prognosegüte besitzt. Diese Verbesserung zeigt sich vor allem in Zeiten ökonomischer Turbulenzen wie der jüngsten Finanzkrise.

Nowcasting GDP with a large factor model space*

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Abstract

We propose a novel time-varying parameters mixed-frequency dynamic factor model which is integrated into a dynamic model averaging framework for macroeconomic nowcasting. Our suggested model can efficiently deal with the nature of the real-time data flow as well as parameter uncertainty and time-varying volatility. In addition, we develop a fast estimation algorithm. This enables us to generate nowcasts based on a large factor model space. We apply the suggested framework to nowcast German GDP. Our recursive out-of-sample forecast evaluation results reveal that our framework is able to generate forecasts superior to those obtained from a naive and more competitive benchmark models. These forecast gains seem to emerge especially during unstable periods, such as the Great Recession, but also remain over more tranquil periods.

Keywords: Dynamic factor model, forecasting, GDP, mixed-frequency, model averaging, time-varying-parameter

JEL classification: C11, C32, C51, C52, C53

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*We thank our reviewer Dimitris Korobilis for his valuable comments and suggestions. We also thank Johannes Bleher, Thomas Dimpfl, Thomas Götz, Malte Knüppel, Giorgio Primiceri, Christian Schumacher, John B. Taylor, Leif A. Thorsrud, Maik Wolters as well as seminar participants at the Deutsche Bundesbank, at the Vienna Workshop on High Dimensional Time Series in Macroeconomics and Finance 2019 and at the Third Research Conference on Macroeconomic Modelling and Model Comparison for their helpful discussions.
This paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank, the European Central Bank or the Eurosystem.

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1 Introduction

Policy makers such as central banks require an assessment of the past, current and future state of economic activity to conduct informed and responsible monetary policy. Actual statistics especially on key economic indicators, such as GDP or unemployment are, however, only available with considerable delay. Therefore the recent nowcasting literature aims at developing macroeconomic models that rely on information that is available in a more timely manner to produce early estimates of GDP and other key indicators alike, when they are not yet available. The indicators' availability at often different frequencies, asynchronous release dates and publication lags as well as changing dynamics of the economy, however, impose challenges on nowcasting models that are a unique feature of this literature and require specifically tailored model solutions.

There is a growing number of studies addressing these specific requirements. Mariano and Murasawa (2003) develop a mixed-frequency dynamic factor model, which is able to deal with missing data due to publication delays and non-uniform data frequencies. Their framework is successfully applied in macroeconomic nowcasting by e.g. Bańbura, Giannone, and Reichlin (2010). Besides the issues implied by the real-time data flow, a changing economic environment can bring about changes in the economic transition mechanism, the comovement of variables, and the nature and heteroskedasticity of exogenous shocks. Against this backdrop, Primiceri (2005) and more recently Thorsrud (2018) introduce time-varying parameters and stochastic volatility into model components of VARs and factor models to reduce their vulnerability against structural breaks. Moreover, not only parameters but also the forecasting model itself might suffer from model breakdown, instability, and change, as the economy undergoes structural changes. Banerjee, Marcellino, and Masten (2005) confirm - at least for the euro area - that the "best" indicator for inflation and GDP growth is changing over time and suggest updating the choice of variables continuously. Working with a single parsimonious model that contains a fixed selection of predictors is hence not advisable. One potential solution to this problem is model averaging or model selection. Raftery, Kárný, and Ettler (2010) introduce dynamic model averaging (DMA), which is a recursive implementation of Bayesian model averaging (BMA) and is able to combat model uncertainty in the context of predicting the output strip thickness of a cold rolling mill in real-time. In the context of macroeconomic forecasting DMA, where at each point in time the weights of models contained in a given model space are updated conditional on past forecast performance, and its extension, dynamic model selection (DMS), where the model with the highest DMA weight is chosen, are successfully applied by Koop and Korobilis (2011), Koop and Onorante (2013) and Onorante and Raftery (2016). They find that these techniques can greatly improve the forecast performance compared to conventional models. While the above mentioned studies address some of the unique requirements of nowcasting models, a unified approach that deals with all aspects simultaneously seems to be missing. We seek to fill this gap in the literature.

We propose a novel time-varying parameters mixed-frequency dynamic factor model (TVP-MF-DFM) which is integrated in a dynamic model averaging framework for macroeconomic nowcasting. In doing so, we contribute to the nowcasting literature in various ways. First, we build a TVP-MF-DFM that can efficiently deal with the properties of the real-time data flow, parameter change and time variation in the volatility. Regarding these characteristics, our model is closely related to that proposed by Thorsrud (2018). Instead of applying standard Bayesian techniques, however, we follow a different estimation strategy. Second, we develop a fast dual one-step Kalman filter algorithm which only requires a single iteration. Therefore, we extend the algorithm proposed by Koop and Korobilis (2014) to account for mixed-frequency data. This algorithm enables us to estimate a large model space in a reasonable amount of time which leads to our third contribution. We estimate our TVP-MF-DFM in a unified dynamic model averaging framework, where nowcasts are based on many different model specifications, which accounts for time-varying forecasting performance. Moreover, our framework is able to generate density forecasts illustrating the uncertainty around the point forecasts. Finally, utilising DMA we can also shed light on the time-varying importance of the economic indicators in our data set and hence on the drivers of nowcasts which might be of paramount interest to policy makers.

In our empirical exercise we apply our TVP-MF-DFM-DMA to nowcast German GDP and compare its forecast performance to a naive as well as more competitive benchmark models. Our recursive out-of-sample forecast evaluation results reveal that our suggested framework can produce forecasts that are superior to those of competing models. These forecast gains seem to emerge especially during unstable periods, such as the Great Recession, but also remain over more tranquil periods. Overall, the proposed model specifications manage to improve the forecast accuracy of the naive benchmark by up to 61%. Compared to more competitive benchmarks, forecast performance improves by up to 40% in the most favourable case. Moreover, along similar lines to Pettenuzzo and Timmermann (2017), we find that accounting for time-varying parameters can indeed improve forecast performance. In our case these improvements are more pronounced for medium length forecast horizons and performance based weighting schemes. Additionally, we find that these more elaborate forecast combinations only seem to perform better than simple averaging schemes for very short forecast horizons, which might be attributed to update lags that result from publication lags of the target variable as well as the length of the forecast horizon itself. Furthermore, our results show that the expected model size fluctuates around twelve indicators on average. This also confirms the findings of Bai and Ng (2008) indicating that "targeted predictors" may improve the forecast accuracy of factor models. Finally, an indicator heatmap visualizes time-varying importance of predictors shedding light on the factor black-box.

The remainder of this paper is organized as follows. The next section introduces the econometric framework and the estimation algorithm. Section 3 provides an overview over the forecast setup. Section 4 presents the empirical results and the final section concludes.

2 Econometric Methodology

2.1 The Modeling Framework

Throughout the following, upper case letters will indicate matrices and lower case letters will indicate vectors. Further, we adopt the convention that a superscript M (Q) indicates model components that are related to monthly (quarterly) variables. Let x_t denote the n dimensional zero-mean vector of stationary variables. We now assume that, at monthly frequency, the economy linearly depends on k latent factors, f_t, that are common to all variables contained in x_t and satisfy $E(f_t u_t) = 0$.

$$x_t = \Lambda_t \cdot f_t + u_t, \qquad u_t \sim N(0, V_t) \tag{1}$$

The so called common component $(\Lambda_t \cdot f_t)$ captures the variability in the dependent variables that is due to the common factors. The idiosyncratic zero-mean Gaussian disturbances u_t capture the remaining variability. We assume them to be cross-sectionally and serially uncorrelated and thus do not model their dynamics. Since Bańbura and Modugno (2014) find that explicitly accounting for serial correlation does not lead to consistent improvements of GDP forecasts, however, we do not expect this simplification to influence our results noticeably. Given it is not realistic to assume that the factors evolve independently over time, we further define a dynamic process for the factors that is given by a pth order VAR

$$f_t = B_{t,1} f_{t-1} + \cdots + B_{t,p} f_{t-p} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, Q_t) \tag{2}$$

where ε_t are again serially uncorrelated zero-mean Gaussian disturbances. It is noteworthy that instead of extracting factors and using them to augment univariate forecasting regressions, we model the variables jointly in a multivariate system. This should improve the identification of the variables' co-movements and thus benefit the forecasting results (also see Koop and Korobilis, 2014).

Empirically, relying on a factor model specification has another appeal. With increasing data availability also macroeconomic forecasting suites have been growing. High dimensional VARs that potentially contain hundreds of variables, however, might suffer from the curse of dimensionality. Factor models on the other hand, by relying on only a few latent factors, can summarize the information contained in large data sets efficiently without running into these issues (see e.g. Giannone, Reichlin, and Small, 2008; Korobilis, 2012; Pirschel and Wolters, 2017). Moreover, Pirschel and Wolters (2017) compare different techniques for extracting the information contained in large data sets and provide evidence that the combination of factor models and shrinkage - which we implement through DMA/DMS - seems to be the most efficient approach.

Given the overwhelming evidence for structural breaks in several U.S. and European macroeconomic time series provided by Cogley and Sargent (2005), Banerjee, Marcellino, and Masten (2008), Breitung and Eickmeier (2011) and Bauwens, Koop, Korobilis, and Rombouts (2015) we further want to equip our dynamic factor model with the necessary flexibility to account for parameter instability. The importance of accounting for structural breaks in factor models and forecasting models in general in order to achieve good forecast performance was documented by Banerjee et al. (2008) and Bauwens et al. (2015), respectively. Generally, changes in the structure of the economy, the monetary policy regime, the conduct of economic policy or technological change can alter the economic transition mechanism, the co-movements of variables, and the nature and heteroskedasticity of exogenous shocks (see e.g. Breitung and Eickmeier, 2011; Primiceri, 2005). In case of the above model, these changes translate to time-variation in the factor loadings, $\Lambda_{t,i}$, the parameter matrices in the dynamic factor process, $B_{t,i}$, and the variance-covariance matrices V_t and Q_t. Since Pettenuzzo and Timmermann (2017) find that TVP models that allow parameters to change gradually over time outperform Markov Switching and

Change Point models and improve density as well as point forecasts, we assume that the vectors of loadings and VAR coefficients evolve as multivariate random walks

$$\lambda_t = \lambda_{t-1} + \upsilon_t, \qquad \upsilon_t \sim N(0, W_t) \tag{3a}$$

$$\beta_t = \beta_{t-1} + \eta_t, \qquad \eta_t \sim N(0, R_t) \tag{3b}$$

where $\lambda_t = vec(\Lambda_t)$ and $\beta_t = (vec(B_{t,1})', \ldots, vec(B_{t,p})')'$. Moreover, υ_t and η_t are serially uncorrelated and feature the time-varying covariance matrices W_t and R_t. All disturbance vectors are further assumed to evolve independently. This completes the description of our basic TVP-DFM.

2.2 Temporal Aggregation

As pointed out above, the available economic indicators are usually observed at different frequencies. While GDP is observed at quarterly frequency, other indicators such as industrial production are available on a monthly basis. It is common practice to pre-filter the data, e.g. by means of temporal aggregation or interpolation, to align the different frequencies in the data. This might, however, destroy important information and induce mis-specification (see Foroni and Marcellino, 2013). Since the model given in equations (1) and (2) is specified at monthly frequency, we follow the nowcasting literature instead and define the relationship that links the quarterly variables to their latent high frequency counterparts. This relationship critically hinges on whether the variables of interest are stock or flow variables and on how they have been transformed before entering the model. In case of a quarterly flow variable such as GDP one has the accounting identity

$$Y_t^Q = Y_t^M + Y_{t-1}^M + Y_{t-2}^M, \tag{4}$$

where Y_t^M denotes the unobserved monthly counterpart of Y_t^Q during the respective quarter. After transforming the observed Y_t^Q by applying log-differences in order to assure stationarity, one can define the partially observed monthly series[1]

$$y_t^Q = \begin{cases} log(Y_t^Q) - log(Y_{t-3}^Q), & t = 3, 6, 9... \\ unobserved, & otherwise, \end{cases} \tag{5}$$

where y_t^Q is observed every third month and unobserved during the first and second month of every quarter (see e.g. Bańbura et al., 2010). Following e.g. Bańbura et al. (2010, 2013) and combining equations (4) and (5), one can now apply the approximations in Mariano and Murasawa (2003, 2010), which yield the final aggregation scheme for the log-differenced flow variable y_t^Q

[1] A more general treatment is provided by e.g. Bańbura, Giannone, Modugno, and Reichlin (2013) and Aruoba, Diebold, and Scotti (2009).

$$
\begin{aligned}
y_t^Q = log(Y_t^Q) - log(Y_{t-3}^Q) &= \sum_{i=0}^{2} log\left(Y_{t-i}^M\right) - \sum_{i=0}^{2} log\left(Y_{t-i-3}^M\right) \\
&\approx y_t + 2y_{t-1} + 3y_{t-2} + 2y_{t-3} + y_{t-4}, \qquad\qquad for \quad t = 3, 6, 9...
\end{aligned}
\tag{6}
$$

where $y_t = \Delta log(Y_t^M)$. As Bańbura et al. (2013) point out, this specification keeps the constraints on the observational relationship linear. The resulting state space can thus be estimated by means of Kalman filters (also see Elliott and Timmermann, 2016).

2.3 A TVP-MF-DFM

After having described the TVP-DFM and the aggregation scheme for the quarterly variables, we can now explore the restrictions which the aggregation scheme imposes on the structure of the factor model. Let y_t^M denote the m variables that are originally observed at monthly frequency and y_t^Q denote the q variables that are only available at quarterly frequency, where Λ_t^M and Λ_t^Q denote the corresponding loading matrices. Combining equation (1) and equation (6) and casting the system into state space form, we have

$$x_t = H_t s_t + u_t, \qquad u_t \sim N(0, V_t) \tag{7a}$$

$$s_t = A_t s_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, Q_t) \tag{7b}$$

$$\lambda_t = \lambda_{t-1} + v_t, \qquad v_t \sim N(0, W_t) \tag{7c}$$

$$\beta_t = \beta_{t-1} + \eta_t, \qquad \eta_t \sim N(0, R_t) \tag{7d}$$

with

$$
x_t = \begin{bmatrix} y_t^M \\ y_t^Q \end{bmatrix}, \qquad H_t = \begin{bmatrix} \Lambda_t^M & 0 & 0 & 0 & 0 & 0_{(m\times p-5)} \\ \Lambda^Q & 2\Lambda^Q & 3\Lambda^Q & 2\Lambda^Q & \Lambda^Q & 0_{(q\times p-5)} \end{bmatrix},
$$

$$
s_t = \begin{bmatrix} f_t & f_{t-1} & \dots & f_{t-4} & \dots & f_{t-p+1} \end{bmatrix}'
$$

with $\Lambda^Q_{(q\times k)}$, $\Lambda^M_{(m\times k)}$ and

$$
A_t = \begin{bmatrix} B_{1,t} & \dots & B_{p-1,t} & B_{p,t} \\ I & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix}, \qquad u_t = \begin{bmatrix} u_{1,t} \\ \vdots \\ u_{N,t} \end{bmatrix}, \qquad \varepsilon_t = \begin{bmatrix} e_t \\ 0 \\ \vdots \end{bmatrix}
$$

where Q_t is singular with a non-singular block in the upper left corner. Furthermore, note that V_t is diagonal, which prevents that there exists a continuum of observationally equivalent models which are defined by arbitrary factor dependencies (see Nakajima and West, 2013). Since the algorithm employed in a later section relies on principal component estimates as starting values for the factors, the model is identified up to a sign rotation without having to impose further restrictions on the loadings matrix (also see Koop and Korobilis, 2014).

The aggregation scheme is absorbed by matrix H_t and implies that the state vector s_t must have at least five elements. Compared to an approach with augmented regressions, this setup thus has the advantage that the monthly as well as the quarterly variables contribute to the factor estimate. Importantly, note that the loadings matrix of the quarterly variables, Λ^Q, is static. This is due to the fact that time-variation in the loadings and the aggregation scheme described by equation (6) are in conflict with each other (see Thorsrud, 2018). Moreover, note that the same aggregation scheme theoretically also applies to the idiosyncratic components of the quarterly variables. To simplify the estimation, however, we follow Bańbura et al. (2013) and Thorsrud (2018) and stick to our original assumption. We thus assume that they are serially and cross-sectionally uncorrelated zero-mean Gaussian disturbances with time-varying covariance matrices at the frequency at which they are defined.

2.4 Estimation Algorithm

Models such as the TVP-VAR of Primiceri (2005) or the TVP-DFM proposed by Del Negro and Otrok (2008) are usually estimated with Bayesian methods involving Markov Chain Monte Carlo (MCMC) algorithms, such as Gibbs samplers.[2] The obvious drawback of this procedure, however, is high computational demand. Adding to this, the Kalman filters and multivariate stochastic volatility models that are usually used to assess the conditional distributions of the time-varying parameters and covariance matrices have to be reestimated at each iteration of the algorithm. The estimation of even a single TVP-MF-DFM by means of MCMC schemes thus comes at the cost of a high computational burden. When faced with a recursive forecasting exercise on an expanding window of data or many different model specifications, as is the case with the model averaging and selection techniques we employ in our empirical exercise, the computational demand again multiplies and quickly becomes unbearable. In a real world scenario, where a policy making institution cannot afford to wait several days or even weeks for a forecast to be produced, this renders MCMC schemes inapplicable in our setup.

Instead, for the parameter estimation of our model we develop a fast dual one-step Kalman filter algorithm building on the TVP-FAVAR algorithm proposed by Koop and Korobilis (2014) that only requires one single iteration. The general idea is to circumvent the need for recursive sampling by conditioning on principal component estimates for the factors during the estimation of the model parameters and by replacing the multivariate stochastic volatility models that are usually used for V_t, Q_t, W_t, and R_t by variance discounting methods (see e.g. Aguilar and West, 1998). Although we stick to using Kalman filters to estimate the time-varying parameters and the factors, it is these changes that break up the recursiveness of the Gibbs algorithm and thus allow for simulation-free estimation. Our algorithm evolves as follows:

TVP-MF-DFM Algorithm

1) Initialization
    (a) Initialize the hyperparameters
    (b) Standardize the data
    (c) Estimate the preliminary factors, f_t^{PC}, by principal components
2) Parameter Estimation given f_t^{PC}

[2] The reader is referred to Blake and Mumtaz (2017) for an introduction to Bayesian estimation methods which are widely used in central banks.

(a) Estimate V_t, Q_t, W_t, and R_t using variance discounting methods
(b) Estimate λ_t^M and β_t conditional on V_t, Q_t, W_t, R_t, and f_t^{PC} using Kalman filters & smoothers
(c) Estimate λ^Q conditional on V_t, Q_t, W_t, R_t, and f_t^{PC} using Kalman filters & smoothers

3) Estimate f_t conditional on the model parameters using a Kalman filter & smoother

Given the mixed-frequency structure of our data, we cannot estimate the principal component factors from x_t directly. Instead, we first linearly interpolate the periodically missing values of the quarterly variables and apply a spline smoother to smooth the resulting step function. The principal component factors, f_t^{PC}, are then estimated jointly from the monthly variables and interpolated quarterly variables.[3] For the algorithm to work well, it is important that these preliminary factors provide a good approximation of the factors produced by our highly flexible TVP-MF-DFM even in the event of structural breaks. Although theoretical proof for our highly nonlinear model is not available, Bates, Plagborg-Møller, Stock, and Watson (2013) show that principal components generally produce consistent factor estimates even with substantial time-variation in the factor loadings, as is the case under the specification in equation (7c).

Since most of our algorithm depends on the Kalman filter, for the reader's convenience we briefly introduce the Kalman filter & smoother equations in Appendix A.1. Using the Kalman filter recursions in equations (20a)-(20f) we estimate the factors, f_t, conditional on the parameters and the dependent variables contained in x_t. One of the many appeals of working in state space and with Kalman filters is the easy treatment of missing values, regardless of whether they occur periodically or at the beginning or end of the sample. This allows for simultaneous treatment of mixed frequencies, the ragged edge or other kinds of missing data. Over the years, the literature has proposed many equivalent approaches. Mariano and Murasawa (2003) propose to impute zeros for y_t^Q when it is missing and to set the corresponding loadings to zero. Giannone et al. (2008) suggests setting the residual variance to infinity instead and Durbin and Koopman (2012) simply view the dimensions of the state space as time-varying. Although we follow Mariano and Murasawa (2003), all of these approaches manipulate the state space and induce the Kalman filter to "skip" missing observations so that they do not contribute to the new value of the state vector and its variance. At every point in time, only the information contained in the observed variables is considered. Finally, we smooth the resulting factor estimates with the Rauch-Tung-Striebel fixed interval smoother (see Haykin, 2001; Rauch, Tung, and Striebel, 1965).

Since the innovations in equation (7a) are independent across the variables in x_t conditional on knowing s_t and assuming that the loadings are uncorrelated across variables, we can sample the loadings equation-by-equation. This allows for separate estimation of λ_t^M and λ^Q. To estimate λ_t^M we simply replace the appropriate elements of equations (20a)-(20f) and equations (21a)-(21c) with their respective counterparts. Most noticeably, since λ_t^M evolves as a random walk, A_t drops out of all equations entirely. The estimation of the static λ^Q is more complicated due to the aggregation scheme in equation (7a) and the high degree of time-variation in the model. Relying on e.g. a Bayesian regression requires

[3] Note that the results reported in a later section are robust against applying a spline smoother or using the linearly interpolated series directly.

knowledge of the variances, V_t^Q. To estimate V_t^Q as outlined below, however, we require an estimate of λ^Q at every point in time. Thus a recursive method to estimate λ^Q is needed. Moreover, the sequence of quarterly variables is only partially observed and thus implies a dependent variable with many missing values. To work around this problem, we develop a simple trick. First, conditional on the factors, we rotate the state space in equation (7a) and factor out Λ^Q. The relationship is now expressed as the product of the quarterly loadings and a moving average of the factors.

$$\begin{aligned} y_t^Q &= \Lambda_t^Q \cdot \sum_{i=0}^{4} \omega_i f_{t-i} + u_t, \qquad u_t \sim N(0, V_t) \\ \omega_i &= \begin{cases} i+1, & for \quad i = 0,1,2 \\ 6-i-1, & for \quad i = 3,4 \end{cases} \end{aligned} \tag{8}$$

To estimate λ^Q, we now manipulate equation (7c) to read

$$\lambda_t^Q = \lambda_{t-1}^Q \tag{9}$$

and estimate the static quarterly loadings recursively by means of a Kalman filter, which is similar to working with recursive OLS. These changes do not change the structure of the Kalman filter equations (20a)-(20f), which simply have to be adapted by replacing the appropriate elements. Whenever y_t^Q is observed, an update of the static parameter given the new information set occurs. When it is unobserved we follow our practice from above and simply do not update λ^Q. In the forgetting factor framework, which is described below, the implementation is particularly simple by setting $\kappa_3 = 1$ for the quarterly variables, which implies that $W_t = 0$. The distribution of the residuals thus collapses on zero. Since λ^Q is static by construction, it always coincides with the last Kalman filter update. As the smoothed λ^Q estimate, we hence simply accept the most recently updated estimate for all time periods. Estimation of the VAR coefficients, β_t, again proceeds as in Koop and Korobilis (2014) and requires a Kalman filter and smoother following equations (20a)-(20f) and (21a)-(21c), where we only accept non-explosive draws. To enforce this restriction, whenever the biggest eigenvalue lies outside the unit circle, we replace the current β_t-update by 0.95 times the previous update. The Kalman filter recursions then proceed as before.

The variance discounting methods are simulation free and imply recursive estimation of V_t, Q_t, W_t, and R_t. For V_t and Q_t we follow Koop and Korobilis (2014) and use exponentially weighted moving average estimators (EWMA). This method is appealing, because EWMA produce minimum mean squared error forecasts (see Muth, 1960) that are equivalent to those produced by simple state space or simple ARIMA models (see Durbin and Koopman, 2012). Moreover, as Koop and Korobilis (2014) point out, EWMA provide an accurate approximation of integrated GARCH models and are thus in line with the features of the macroeconomic VAR literature that usually works with integrated stochastic volatility models (see e.g. Primiceri, 2005). Despite relying on this rather simple algorithm, this allows to stay as close as possible to the standard methods. Equations (10a, 10b) represent the EWMA estimator, where $u_{i,t}$ and $e_{i,t}$ are backed out according to equations (7a) and (7b). With slight abuse of notation, $Q_t(k)$ denotes the k_{th} leading principal submatrix of Q_t and $diag(u_t u_t')$ denotes the diagonal matrix of the variance covariance matrix of the residuals u_t. We use the diagonal matrix in equation (10a) to enforce the standard identifying constraint spelled out in section 2.3 and use the k_{th} leading principal submatrix in equation (10b) to restrict the EWMA to the nonsingular block of Q_t.

$$V_t = \kappa_1 V_{t-1} + (1-\kappa_1) \cdot diag(u_t u_t') \tag{10a}$$
$$Q_t(k) = \kappa_2 Q_{t-1}(k) + (1-\kappa_2) e_t e_t' \tag{10b}$$

The degree of time-variation in V_t and Q_t is governed by the two decay parameters κ_1 and κ_2, respectively. As pointed out above, the mixed-frequency structure of our data introduces periodically missing values in the quarterly variables. Thus, the residuals necessary to compute the EWMA for V_t^Q are only available at the end of each quarter. We hence follow West and Harrison (1997), who provide suggestions on the treatment of missing values in EWMA and only update when actual data is available. Throughout the quarter, the EWMA does not decay and remains at its value.[4] This update lag, however, results in slower time-variation of V_t^Q than V_t^M even given the same κ_1. To compensate for this effect, in our empirical exercise we allow V_t^Q and V_t^M to change at different rates and introduce the decay parameters κ_1^Q and κ_1^M. Finally we follow Koop and Korobilis (2014) and produce smoothed estimates of V_t^Q, V_t^M, and Q_t.

For W_t and R_t we use the forgetting factor methods described in Raftery et al. (2010) and Koop and Korobilis (2012) and thus estimate these matrices directly from the respective state covariance matrix estimate provided by the Kalman filter. From standard Kalman filter inference, we know that λ_t and β_t in equations (7c) and (7d) are given by

$$\lambda_t | Data_{1:t-1} \sim N(\lambda_{t|t-1}, \Sigma^{\lambda}_{t|t-1}) \tag{11a}$$
$$\beta_t | Data_{1:t-1} \sim N(\beta_{t|t-1}, \Sigma^{\beta}_{t|t-1}) \tag{11b}$$

where by equations (7c) and (7d)

$$\Sigma^{\lambda}_{t|t-1} = \Sigma^{\lambda}_{t-1|t-1} + W_t \tag{12a}$$
$$\Sigma^{\beta}_{t|t-1} = \Sigma^{\beta}_{t-1|t-1} + R_t \tag{12b}$$

Following Raftery et al. (2010) and Koop and Korobilis (2014) one can now define $W_t = (\kappa_3^{-1} - 1)\Sigma^{\lambda}_{t-1|t-1}$ and $R_t = (\kappa_4^{-1} - 1)\Sigma^{\beta}_{t-1|t-1}$ to replace equations (12a) and (12b) by

$$\Sigma^{\lambda}_{t|t-1} = \kappa_3^{-1} \Sigma^{\lambda}_{t-1|t-1} \tag{13a}$$
$$\Sigma^{\beta}_{t|t-1} = \kappa_4^{-1} \Sigma^{\beta}_{t-1|t-1} \tag{13b}$$

[4] Since the Kalman filters treat missing information similarly, this also has the appeal of maintaining a consistent approach to the treatment of missing data.

which introduces the forgetting factors, κ_3 and κ_4, respectively. Hence the model is still a properly defined state space and the Kalman filters and smoothers proceed in standard fashion. The interpretation of the decay parameters and forgetting factors is generally the same. Lower values put lower weight on past observations and thus allow for faster parameter change. A value of one, however, implies constant parameters. In our empirical application we use this feature to estimate nested MF-DFM models to assess the merits of allowing for time-variation.

The smoothed estimates for V_t, Q_t, λ_t^M, λ^Q, β_t, and f_t take effect throughout different steps of our algorithm and empirical exercise, respectively. In scope of this, we rely on the smoothed estimates of V_t, Q_t, λ_t^M, λ^Q, and β_t when estimating and smoothing the factors f_t. These estimates thus benefit directly from the smoothed series. Our empirical exercise - that we introduce in a later section - reaps an additional benefit from the smoothed time series. Given that future information is available when backcasts are produced, relying on smoothed estimates instead of the Kalman filter results alone allows for exploiting future information that would otherwise be neglected.

2.5 Dynamic Model Averaging

Given an arbitrary forecasting suite, we are faced with a multitude of TVP-MF-DFM of different sizes,

$$x_t^{(j)} = H_t^{(j)} s_t^{(j)} + u_t^{(j)}, \qquad u_t^{(j)} \sim N(0, V_t^{(j)}) \tag{14a}$$
$$s_t^{(j)} = A_t^{(j)} s_{t-1}^{(j)} + \varepsilon_t^{(j)}, \qquad \varepsilon_t^{(j)} \sim N(0, Q_t^{(j)}) \tag{14b}$$
$$\lambda_t^{(j)} = \lambda_{t-1}^{(j)} + v_t^{(j)}, \qquad v_t^{(j)} \sim N(0, W_t^{(j)}) \tag{14c}$$
$$\beta_t^{(j)} = \beta_{t-1}^{(j)} + \eta_t^{(j)}, \qquad \eta_t^{(j)} \sim N(0, R_t^{(j)}) \tag{14d}$$

where j is a single model from a model space M. Each of the $j \in [1, \ldots, J]$ models features a different subset of variables. Given the number of factors, k, this amounts to at most 2^{n-k} different variable combinations and hence candidate forecasting models that could be appropriate at every point in time.

One could of course resort to working with the model containing all predictors or could attempt to find a single parsimonious model that beats its alternatives conditional on the sample. As pointed out above, structural changes in the economy, however, are likely to cause model uncertainty, breakdown, and change. While some economic indicators might predict GDP particularly well during phases of stable growth, they might lose predictive power during e.g. economic crises and vice versa. Moreover, the same factors that drive parameter change, such as policy decisions, the demise of industrial sectors or technological advancement, might alter the forecast performance of economic predictors naturally over time. Banerjee et al. (2005) confirm for euro area inflation and GDP growth that the "best" predictor is changing over time and suggest updating the choice of variables continuously. Although it might be possible to find a single "best" parsimonious model, given these findings it is likely to be rather dependent on the sample and can thus not be expected to beat all its alternatives forever. Using the entire set of predictors in a single model instead, is in turn likely to induce over-parameterization, which again deteriorates the out-of-sample forecast performance. No single model with a constant set of predictors can thus be expected to beat all other models at every point in time (also see Banerjee et al., 2005).

One possible solution is model selection or model averaging, where in this setup one would like those methods to be dynamic and computationally cheap. Given the uncertainty surrounding either the "best" model or the "optimal" forecast combination of the models contained in M, this poses two major challenges. First, one requires an assessment of the probability that model j applies at time t. Second, conditional on these individual model probabilities, one needs to either select a model from M or produce a forecast combination of all models contained in the model space. The time dependence is what makes this procedure dynamic, where the forecast combinations or the selected models depend on t instead of the entire sample. Raftery et al. (2010) propose a recursive updating method called dynamic model averaging (DMA) that is designed to do exactly that and can be applied as a wrapper method for state space models. Koop and Korobilis (2011, 2012, 2014) and Koop and Onorante (2013) have already successfully applied this framework to the economics literature and find that DMA can improve forecast performance. The appeal of this method is that model averaging/selection is conducted conditional on the individual past forecast performance of every model. Additionally, by assigning weights to individual forecasts and thus indirectly to the models' parameters, DMA/DMS implies shrinkage, which serves to counteract possible over-parameterization especially in larger models (see Koop and Korobilis, 2011). In a factor model context, the different sets of variables that form the models contained in M further imply that DMA/DMS average or select different factor structures. Given the findings of Boivin and Ng (2006) and Bańbura and Modugno (2014), who provide evidence that factor extraction might actually suffer when data is added that provides little information about the factors and increases the noise-to-signal ratio, this has an additional benefit. In our TVP-MF-DFM context, DMA/DMS can give more weight to models that combine predictors that provide more information on the factors and thus facilitate factor extraction. When a model is based on a dataset with a high noise-to-signal ratio instead, it can simply assign lower weights. DMS/DMA thus leave it to the data to decide which model or how much of each model - and with it the implied factor structure - is desirable at each point in time.

To motivate their framework, Raftery et al. (2010) assume a hidden Markov chain for the model space, where the state of the system itself depends on the current value of the chain. This is, what allows the "best" model to vary over time. Standard techniques for the estimation of hidden Markov models, such as particle filters or ensemble Kalman filters, are computationally demanding and thus unfeasible in this application. The fact that all J models can be estimated independently with the algorithm proposed in the previous subsection, however, allows for a very simple approximation that involves updating the models' probabilities individually at each point in time (see Raftery et al., 2010).

In the general framework, $\pi_{t|t-1,j}$ usually denotes the probability that model j applies at time t, conditional on the information set valid through $t-1$. Since it is not reasonable to believe that this probability is independent of the forecast horizon h, we extend the general framework and define $\pi_{t|t-1,j,h}$. Given that some variables might e.g. be more informative for longer forecast horizons, while others are more suitable short-term predictors, this allows the models to "specialize" on forecast horizons. In the DMA/DMS framework, the models that work best at forecast horizon h, can receive higher probabilities than models that work best at other horizons. As pointed out above, one now needs to define how the individual model probabilities evolve through time. Raftery et al. (2010) propose the following approximations,

$$\pi_{t|t-1,j,h} = \frac{\pi^{\gamma}_{t-1|t-1,j,h}}{\sum_{k=1}^{J} \pi^{\gamma}_{t-1|t-1,k,h}} \tag{15}$$

$$\pi_{t|t,j,h} = \frac{\pi_{t|t-1,j,h} PL_j(Data_t|Data_{1:t-h})}{\sum_{k=1}^{J} \pi_{t|t-1,k,h} PL_k(Data_t|Data_{1:t-h})} \tag{16}$$

where equation (15) is the prediction step and equation (16) is the update step. $\gamma \in [0, 1]$ is another forgetting factor that controls the rate of time-variation in the individual model probabilities and thus model change. A higher (lower) γ implies slower (faster) model change. Note that for $\gamma = 1$ and $\gamma = 0$ the model probabilities are analogous to those under Bayesian model averaging (BMA) and the simple average, respectively. The predictive likelihood of model j at time t is denoted by $PL_j(Data_t|Data_{1:t-h})$ and serves as a measure of fit for model j (see Koop and Korobilis, 2014). The model probabilities thus evolve conditional on the past forecast performance of the individual models at time t and for forecast horizon h. Dynamic model selection (DMS) forecasts now result if the forecast of the model with the highest individual probability at time t is selected. DMA forecasts arise as the model probability weighted average of the J individual model forecasts.

3 Forecast Setup

3.1 Dataset

We use 20 monthly and quarterly variables for nowcasting GDP in Germany. Alongside the indicators of real economic activity such as industrial production and unemployment, we also include sentiment indicators, such as surveys, and financial indicators like the 10 year government bond yield or the corporate spread, where the latter also serves as a risk measure. Global oil production is included to serve as a measure proxy for world-wide economic activity. In addition to the target variable of our forecasting exercise, German GDP, we also include quarterly euro area and French GDP, because they are more timely available and might thus contribute important information. These three variables compose the block of quarterly variables. All variables enter the model without lag. The dataset is downloaded from the Deutsche Bundesbank's time series database and covers a sample period from January 1991 to June 2019 depending on the publication lag of selected indicators. Table 1 provides an overview of these variables as well as of their frequency, transformation and publication lags.

Although this data set seems somewhat small, Bańbura and Modugno (2014) find that information on the total economy is sufficient for obtaining accurate forecasts of GDP. They compare the forecast performance of a small MF-DFM containing 14 variables, a medium sized variant containing 46 variables, and a large variant containing 101 variables on a euro area data set. While the former two perform comparably well, the latter performs slightly worse. This is in line with new evidence that suggests that more variables

Table 1: Dataset and real-time data flow

Indicator	Freq.	Adj.	Tra.	1st Obs.	RE$_{d7}$	RE$_{d27}$
CPI Inflation	M	1	1	Jan. 92	1 W	3 W
Unemployment	M	1	2	Dec. 91	1 W	3 W
Foreign Orders Automotive Ind.	M	1	2	Jan. 91	5 W	7 W
New Orders Manufacturing Ind.	M	1	2	Jan. 91	5 W	7 W
Industrial Production	M	1	2	Jan. 91	5 W	7 W
Nominal Exports	M	1	2	Jan. 91	5 W	7 W
Global Oil Prod. barrels per day	M	0	2	Jan. 91	13 W	11 W
Production Building Industry	M	1	2	Jan. 91	5 W	7 W
EURIBOR	M	2	1	Jan. 91	1 W	3 W
10y Gov. Bond Yield (GER)	M	0	1	Jan. 91	1 W	3 W
Corporate Spread	M	0	2	Jan. 91	1 W	3 W
ifo Business Expectations	M	1	3	Jan. 91	1 W	-
ifo Business Climate	M	0	3	Jan. 91	1 W	-
ifo Ass. of Orders on Hand	M	1	3	Jan. 91	1 W	-
ifo 3 Month Production Plans	M	1	3	Jan. 91	1 W	-
ifo 3 Month Export Expectation	M	1	3	Jan. 91	1 W	-
GfK Consumer Climate	M	0	1	Jan. 91	-	-
GDP (France)	Q	1,3	2	Q1 91	13,5,9 W	15,7,11 W
GDP (Euro Area)	Q	1,3	2	Q1 95	13,5,9 W	15,7,11 W
GDP (GER)	Q	1,3	2	Q1 91	13,17,9 W	15,7,11 W

Notes: This tables gives an overview of time series used in our study. The first column displays the indicator, while its frequency (Freq.) which can be either monthly (M) or quarterly (Q) is given in the next one. Adjustments are given in the third column, denoted as Adj., such as no adjustment (0), calender and seasonally adjusted (1), monthly average (2) and chain linked volume, rebased (3). Data transformations (Tra.) are defined as 1^{st} differences (1), 1^{st} log differences (2) and rebased in order to take the 1^{st} log difference (3). The publication lag of each variables on forecast dates are given in the last two columns. Due to the quarterly frequency of the GDP data the ragged-edge structure of our dataset depends on the forecast date. Accordingly, the numbers in GDP series correspond to the ragged-edge in first, second and third month of each quarter. Note that EURIBOR is replaced with monthly averages of money market rates reported by Frankfurt banks prior to 1999.

do not always benefit factor extraction, especially when the additional variables contain little information about the factors (see Boivin and Ng, 2006). Conversely, so-called targeted predictors can not only reduce the computational burden but also improve the accuracy of forecasts obtained from diffusion indexes and factor models (see Bai and Ng, 2008).

3.2 Forecast Setup

We extend the current DMA literature and evaluate the performance of our model by means of a pseudo-real time recursive out-of-sample forecasting exercise. Since we do not work with historical data vintages, the effect of data revisions on our results will remain unclear. Schumacher and Breitung (2008), however, show that revisions do not affect forecast accuracy considerably such that major changes should not be expected. In order to incorporate various business cycle phases, the evaluation period expands from the first quarter 2007 to the first quarter 2019. Following suit with common central bank practice, we generate our forecasts at the beginning (7^{th}) and the end of the month (27^{th}). The final two columns of Table 1 provide an overview of the ragged-edge on both forecasting dates, where the age of information at the time the forecasts are generated is given in weeks. Throughout the evaluation period we replicate the ragged-edge every month and then apply the algorithms outlined in the previous sections. Starting 29 weeks before the end of the reference quarter, each month we compute a two quarter ahead forecast, a one quarter ahead forecast, and a nowcast to provide an assessment of the future and current state of the economy, respectively. Since German GDP is only released 45 days after the end of the quarter, we also compute backcasts when it is not yet available. This amounts to one backcast in the first month of every quarter when forecasting on the 27^{th} and one backcast each during the first and second month of every quarter when forecasting on the 7^{th}. Altogether, this amounts to three two quarter ahead forecasts, six one quarter ahead forecasts, six nowcasts, and three backcasts. In total we are thus faced with 18 forecast horizons.

To reduce the computational burden and to exclude the forecasting models where no factors are extracted, we restrict German GDP, industrial production, ifo business expectations, and the orders in the manufacturing industry to be contained in every model when doing DMA/DMS. We are thus faced with $2^{16} = 65,536$ candidate forecasting models that span the model space.[5] Generally, we compute no-change forecasts meaning that we compute the forecasts directly conditional on the last estimate of our time-varying parameters instead of simulating their paths. Nonetheless, the forecasts benefit from allowing for time-variation, which is most obvious when illustrated by means of a rather simple example. Let us assume a time series with a single structural break or two regimes, where one regime features a slightly positive and the other one features a slightly negative coefficient. When not allowing for time-varying parameters, the parameter estimate will be a hybrid of both regimes or an average of the appropriate time-varying parameters. A forecasting model based on an actually good predictor might thus nonetheless produce rather poor forecasts (also see Elliott and Timmermann, 2016). In addition, in the above example, the predictor might erroneously be identified as uninformative, due to the hybrid estimate being close to zero. Even though we fix the time-varying parameters at their last estimate, given our selection of forecast horizons and assuming a reasonable degree of time-variation in the parameters of the macroeconomic time series, these parameter estimates are more appropriate than a hybrid estimate across regimes and the general benefit of allowing for time-varying parameters still applies. Despite we only generate point forecasts, forecast densities arise as the densities over the individual model forecasts of all models, where the variance can be interpreted as forecast disagreement. In addition to DMA and DMS we will also compute the two nested cases, the simple average and BMA, as well as the median forecast.

[5] To put that into perspective, a full recursive out-of-sample exercise as the one described roughly takes five days to evaluate on our computers.

3.3 Priors & Starting Values

Our baseline TVP-MF-DFM specification contains three factors, where we allow for five lags in the factor equation. For our prior settings we follow Thorsrud (2018) and set relatively informative priors for the variances and uninformative priors for the remaining parameters,

$$
\begin{aligned}
f_0 &\sim N(0, 10) && \text{(17a)}\\
\lambda_0^M &\sim N(0, 1 \cdot I_n) && \text{(17b)}\\
\lambda_0^Q &\sim N(0, 1 \cdot I_n) && \text{(17c)}\\
\beta_0 &\sim N(\mu_{Min}, \sigma_{Min}) && \text{(17d)}\\
V_0 &\equiv 0.1 \cdot I_n && \text{(17e)}\\
Q_0 &\equiv 0.1 \cdot I_k && \text{(17f)}\\
\pi_{0|0,j,h} &= \frac{1}{J}, \qquad for \quad j = 1, \ldots, J && \text{(17g)}
\end{aligned}
$$

where μ_{Min} and σ_{Min} indicate a Minnesota prior. The idea behind the Minnesota prior is to express beliefs about the structure of the VAR for the factor state equation, where more distant lags are penalized more strongly. We assume that the factor VAR follows a relatively persistent $AR(1)$ with a coefficient of 0.9. The variance is given as $\sigma_{Min} = 0.1/r^2$ for the coefficient on lag r, which corresponds to the choice of Koop and Korobilis (2014). In our empirical application, as a robustness check we will also estimate TVP-MF-DFM featuring one factor and two factors, respectively, that adopt the same prior specifications.

What remains is the specification of the decay parameters and forgetting factors. Since to the best of our knowledge such a model has not been estimated in this framework before, we use a relatively simple grid search to guide our choice of parameters. First, we restrict that κ_1^M and κ_2 as well as κ_3 and κ_4 change at the same rate. This seems reasonable, because both pairs are defined at the same frequency and govern the parameter change of similar components. κ_1^Q is allowed to change at its own rate, for reasons discussed above. We are thus left with a three-dimensional grid, which is given in Appendix A.2. Since we want to avoid data mining issues and a grid search over all models in M would imply an unbearable computational demand, we evaluate the grid points only for the model specification that contains all predictors. This should result in parameter settings that are somewhat optimal for all variables in the data set, whereas evaluating the grid for the smallest specification might result in values that are too specific. Given that our main interest is in point nowcasts, we then adopt the grid point as our parameter specification that minimizes the average mean absolute deviation (MAD) over the nowcast and backcast horizons.[6] To give neither model a head-start we repeat this procedure for both ragged-edges and the one-factor and two-factor TVP-MF-DFM, respectively. This results in the parameter values given in Table 2. For our benchmark models without time-variation, all κ are equal to one, which is why a grid search is not required in this case.

One can make three major observations. First, as suspected, it benefits the forecast performance, if V^Q and V^M are allowed to feature different degrees of time-variation e.g.

[6] Note that the procedure is relatively robust to minimizing the RMSE instead.

Table 2: Decay parameters and forgetting factors

Decay Parameter/Forgetting Factor:	κ_1^M, κ_2	κ_1^Q	κ_3, κ_4
d7:			
TVP-MF-DFM(1)	0.80	0.60	0.99
TVP-MF-DFM(2)	0.99	0.60	0.96
TVP-MF-DFM(3)	0.85	0.60	0.97
d27:			
TVP-MF-DFM(1)	0.60	0.60	0.99
TVP-MF-DFM(2)	0.99	0.60	0.97
TVP-MF-DFM(3)	0.90	0.60	0.97

Notes: d7 indicates the first forecasting date, where forecasts are generated on the 7^{th} of each month. d27 indicates the second forecasting date, where forecasts are generated on the 27^{th} of each month. The number in parentheses indicates the number of factors, k.

to compensate for the update lag that is induced by the missing values of the quarterly variables. Second, the grid search suggests that there is a larger degree of time-variation in the variances than in the time-varying parameters, λ_t and β_t, which is in line with the findings of Koop and Korobilis (2014). Finally, the different shape of the ragged edge on the two forecasting dates seems to have an impact on the choice of the decay parameters and forgetting factors. This might be attributed to the fact that we do not allow for variable specific decay factors. Recalling the way the Kalman filter deals with missing information, we have variables that do not contribute to the estimate of the new state on the 7^{th} but suddenly contribute when forecasting on the 27^{th}. In this case, these variables have an aggravated impact on forecast performance and thus the MAD that are used to optimize the decay parameters and forgetting factors jointly for all variables. Assuming that the different variables demand different rates of parameter switching, the grid search will then produce results that are closer to the switching rate that is more optimal for the variables that are available conditional on the ragged-edge. This then alters the parameter specification.

The forgetting factor γ that governs the model switching rate is not evaluated on a grid and set to the value 0.9, which implies that the forecast performance one year ago only receives 65% of weight. We set this slightly more aggressive value than Koop and Korobilis (2014) for two simple reasons. First, with increasing length of the forecast horizon, more time has to pass until a forecast can be evaluated and used to update the models' weights. Second, the publication lags of the individual variables do not only have to be taken into account when generating the forecasts but also have to be considered when calculating the weights. Since German GDP has a publication lag of 45 days, evaluations of the forecasts for the preceding quarter are thus also only available with delay and not during the first month of a given quarter. This additionally adds considerable update lag. A one quarter ahead forecast that is generated during the first month of a given quarter will thus be weighted with a DMA weight that was updated conditional on the forecast performance of a one quarter ahead forecast generated three quarters ago. For the two quarter ahead forecasts, this update lag increases accordingly. The aggressive γ specification is thus chosen to counteract these two effects. A sensitivity analysis is provided in Appendix

A.3.

4 Forecasting Results

4.1 Forecast Evaluation

Table 3 displays the forecast performance of our baseline model with three factors and time-varying parameters for all weighting schemes and forecast horizons as well as the performance of the corresponding model without time-variation. In addition, we report the forecast performance of the mean forecasts obtained from around 70 model specifications of the Factor MIDAS model (F-MIDAS) developed by Marcellino and Schumacher (2010) for a selection of forecast horizons. This rather competitive benchmark is attractive for two main reasons. First, without the means to average over an entire model space, one usually averages over a few model specifications, if at all. Usually, these model specifications are characterized by either different hyperparameters or a different number of factors. Second, F-MIDAS is also a factor model that provides the means to successfully deal with asynchronous release dates and mixed frequencies. As a second competitive benchmark we use the MF-VAR model proposed by Schorfheide and Song (2015). This mid-sized mixed-frequency VAR is estimated on a comparable data set and belongs to a popular class of mixed-frequency models. These benchmarks thus share the main features required for nowcasting with our TVP-MF-DFM. To evaluate the quality of our point forecasts we provide the MAD and RMSE. All forecast performance measures are given relative to the naive benchmark, which is the in-sample mean.[7] The asterisks indicate that the improvement in forecast accuracy over the benchmark is statistically significant according to the Diebold and Mariano (1995) test.[8] The corresponding tables for a variant with one and two factors, respectively, are provided in Appendix A.3.

The mean and the median forecasts with time-variation and the MF-VAR forecasts are superior to the naive benchmark from the first forecast horizon onwards and remain informative for shorter forecast horizons. It takes DMA, BMA and DMS slightly longer and F-MIDAS the longest to stay continuously informative. Generally, the forecast performance gains over the naive benchmark become increasingly large as time proceeds, where at $h = -5$ BMA with time-variation features a 61% (46%) smaller RMSE (MAD). From 21 weeks prior to the reference quarter onwards, these forecast gains are also statistically significant for some forecast combinations and from $h = 5$ onwards they remain statistically significant for all TVP-MF-DFM variants. Comparing TVP-MF-DFM and F-MIDAS, the TVP-MF-DFM produce relative forecast errors that are smaller than those of F-MIDAS for most periods. Especially for the nowcast and backcast horizon, the relative forecast errors are 10 or more percentage points smaller across all forecast combinations. In this context, the highest relative performance gains again emerge for BMA and the last backcast horizon, where the RMSE (MAD) is reduced by about 35% (20%). Although this holds only true in that particular case, when pooled together the proposed specifications still manage to improve the forecast accuracy of F-MIDAS by 11% (8%) on average in term

7 Since Marcellino and Schumacher (2010) already evaluate the performance of F-MIDAS against various other benchmark models, we do not consider additional benchmarks here.

8 The stationarity of the forecast errors may be violated by the recursive estimation scheme and thus these test results have to be considered cautiously.

Table 3: Relative forecast performance, 3 Factors, $\gamma = 0.9$

h=	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	-1	-3	-5
Relative MAD																		
TVP-MF-DFM Mean	1.00	0.99	0.96	0.97	0.87**	0.86**	0.88*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.64**	0.59**	0.59**	0.56***
TVP-MF-DFM Median	1.00	0.98	0.96	0.98	0.87**	0.86**	0.89*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.65**	0.59**	0.59**	0.57***
TVP-MF-DFM DMS	1.01	1.09	1.06	1.04	0.94	0.95	0.93	0.96	0.94	0.90	0.91	0.87	0.81*	0.66**	0.69**	0.65*	0.60**	0.58**
TVP-MF-DFM DMA	1.01	1.03	1.00	1.05	0.91*	0.93	0.91	0.90*	0.88*	0.87*	0.88*	0.83*	0.81*	0.65**	0.66**	0.59**	0.58**	0.55**
TVP-MF-DFM BMA	1.02	1.04	1.00	1.05	0.91	0.91	0.91*	0.90*	0.89	0.87*	0.87*	0.80**	0.77**	0.63**	0.64**	0.57**	0.59**	0.54**
MF-DFM Mean	0.92*	0.92**	0.90*	0.92	0.87**	0.84***	0.98	0.94	0.91	0.96	1.01	0.90	0.86	0.64**	0.63**	0.58**	0.59**	0.54***
MF-DFM Median	0.92*	0.92**	0.89*	0.92	0.87**	0.84***	1.00	0.94	0.92	0.96	1.00	0.90	0.86	0.64**	0.63**	0.59**	0.59**	0.54**
MF-DFM DMS	0.98	1.01	0.96	0.98	1.00	1.01	0.87**	1.09	1.01	1.14	1.16	0.94	1.15	0.73*	0.69*	0.64*	0.67*	0.52**
MF-DFM DMA	0.95	0.97	0.90*	0.93	0.94	0.92	0.94	1.07	0.98	1.13	1.14	0.96	1.08	0.70*	0.67**	0.61**	0.63*	0.51**
MF-DFM BMA	0.94	0.90**	0.87**	0.93	0.92	0.94	0.94	1.06	0.98	1.19	1.16	0.92	1.02	0.67**	0.64**	0.69*	0.62*	0.54**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.51*	0.45*	0.46*	0.44*
TVP-MF-DFM Median	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.50*	0.45*	0.45*	0.43*
TVP-MF-DFM DMS	0.98	1.04	1.05	1.04	0.95	0.96	0.83	0.91	0.83	0.79	0.77	0.72	0.78*	0.53*	0.52*	0.45*	0.43*	0.40*
TVP-MF-DFM DMA	0.96	1.00	0.99	1.03	0.92*	0.94	0.81	0.82*	0.79	0.77	0.77*	0.70	0.75*	0.49*	0.51*	0.40*	0.40*	0.39*
TVP-MF-DFM BMA	0.96	1.00	0.99	1.04	0.92*	0.95	0.81	0.84*	0.80*	0.76	0.76*	0.69*	0.71*	0.47*	0.48*	0.40*	0.41*	0.39*
MF-DFM Mean	0.93**	0.94**	0.92**	0.95*	0.90**	0.89**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.67	0.47*	0.46*	0.42*	0.43*	0.42*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.90**	0.88**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.68	0.47*	0.47*	0.43*	0.43*	0.42*
MF-DFM DMS	1.01	0.95	0.92	1.03	1.02	0.96	0.85	1.05	0.91	0.95	0.88	0.79	0.97	0.50	0.48	0.42*	0.43*	0.37*
MF-DFM DMA	1.00	0.93	0.91*	1.00	0.98	0.93*	0.86	1.04	0.88	0.94	0.86	0.79	0.93	0.49*	0.48*	0.40*	0.40*	0.36*
MF-DFM BMA	1.00	0.89*	0.88*	1.02	0.94*	0.96	0.86	1.05	0.87	0.96	0.86	0.78	0.83	0.47*	0.47*	0.44*	0.40*	0.39*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: The forecast horizon h is given as the distance to the end of the reference quarter measured in weeks. Negative values indicate backcasts. The benchmark is the in-sample mean. ***, ** and * indicate one-sided Diebold-Mariano p-values that are smaller or equal to 0.01, 0.05 and 0.1, respectively.

of the RMSE (MAD). Compared to the MF-VAR, the same pattern generally emerges, where our proposed model improves upon the forecast accuracy of MF-VAR by up to 43% (26%) and 8% (4%) on average. Interestingly, to a lesser extend the above also holds true for the MF-DFM forecasts. This might indicate an advantage of our method over F-MIDAS and MF-VAR. Unlike the former, instead of linking factors that are extracted at monthly frequency to quarterly GDP by means of the MIDAS equations, we account for quarterly and monthly variables and their interactions directly during the factor extraction which might benefit the forecast performance. Unlike the latter, we efficiently summarize the information contained in the data set in factors, instead of modelling the dynamics across variables separately.

Comparing the TVP-MF-DFM and the MF-DFM models directly, a few interesting features become apparent. First, we find that accounting for time-variation can improve or at least does not harm forecast performance. Our findings are hence in line with those of Bauwens et al. (2015) and Pettenuzzo and Timmermann (2017). Second, these gains seem more pronounced for longer forecast horizons as well as the performance based weighting schemes. Pettenuzzo and Timmermann (2017) find that TVP models improve density forecasts more than they improve point forecasts. Given that the individual model weights are calculated based on the predictive likelihood, which assesses the entire predictive distribution, accounting for time-variation might thus also benefit the calculation of the individual model probabilities and thus forecasts generated with performance based weighting schemes.

Throughout the sequence of forecasts horizons, the performance based weighting schemes do not seem to produce forecasts that are substantially superior to those produced by either the mean or median and thus simple model averaging schemes. This is in line with the findings of Aiolfi, Capistrán, and Timmermann (2010), who compare the performance of different model averaging schemes for several macroeconomic variables and forecasting horizons. Nevertheless, we observe that forecasts from performance based weighting schemes seem to be less precise relative to mean and meadian forecasts for longer horizons but seem to produce slightly more precise forecasts as the forecast horizon decreases. In our baseline model, this holds true for DMA and BMA and the late backcast horizons. As discussed above, this behavior might be rooted in the update lag of the model weights that results due to the nature of forecasting and the publication lag of German GDP. As the forecast horizons get shorter (longer), the update lag decreases (increases) and forecasts from performance based weighting schemes hence become more (less) precise than forecasts from simple weighting schemes. Additionally, one can observe that the forecast performance of DMS is slightly more volatile over the forecast horizons. As discussed above, DMS forecasts are generated conditional on the DMA weights, which in turn depend on the decay factor γ. An optimal γ for DMA does, however, not necessarily imply an appropriate switching rate in the context of DMS. Allowing for two separate γ, one for DMA and one for DMS, might hence improve the performance of DMS.

Lastly, a few final comments are in order. Comparing the performance across the different factor specifications, one can observe that the performance of our model increases in the number of factors for shorter forecast horizons, whereas the models with fewer factors seem to even improve on the performance of the three factor model for longer term forecasts. The performance across these model variants generally appears stable, with the one and two factor model also improving on the forecasting accuracy of our naive as well as more competitive benchmarks across forecast horizons. Comparing the performance across different rates of model switching instead, one can observe that faster rates of model switching seem to benefit longer term forecasts, while our rather slow rate of model switching is superior for shorter forecast horizons. This again underpins our observations regarding the update lag. Finally, one can observe that the forecast performance of all model specifications, F-MIDAS and to a smaller degree MF-VAR feature an erratic pattern over the forecast horizons and thus forecast dates. For the nowcast and backcast horizons, forecasts generated on the 27^{th} seem to perform worse than forecasts generated on the 7^{th}, despite more data and hence information is available. This might indicate that certain data releases alter or disrupt the factor structure to an extend where the forecast performance suffers. Some indicators such as surveys might have leading properties and might thus provide more information on the subsequent than the current quarter (see e.g. Carstensen, Heinrich, Reif, and Wolters, 2017). Including them contemporaneously instead of with lag might then negatively impact the factor extraction and hence lead to such an erratic pattern.

4.2 Shedding Light on the Factor Blackbox

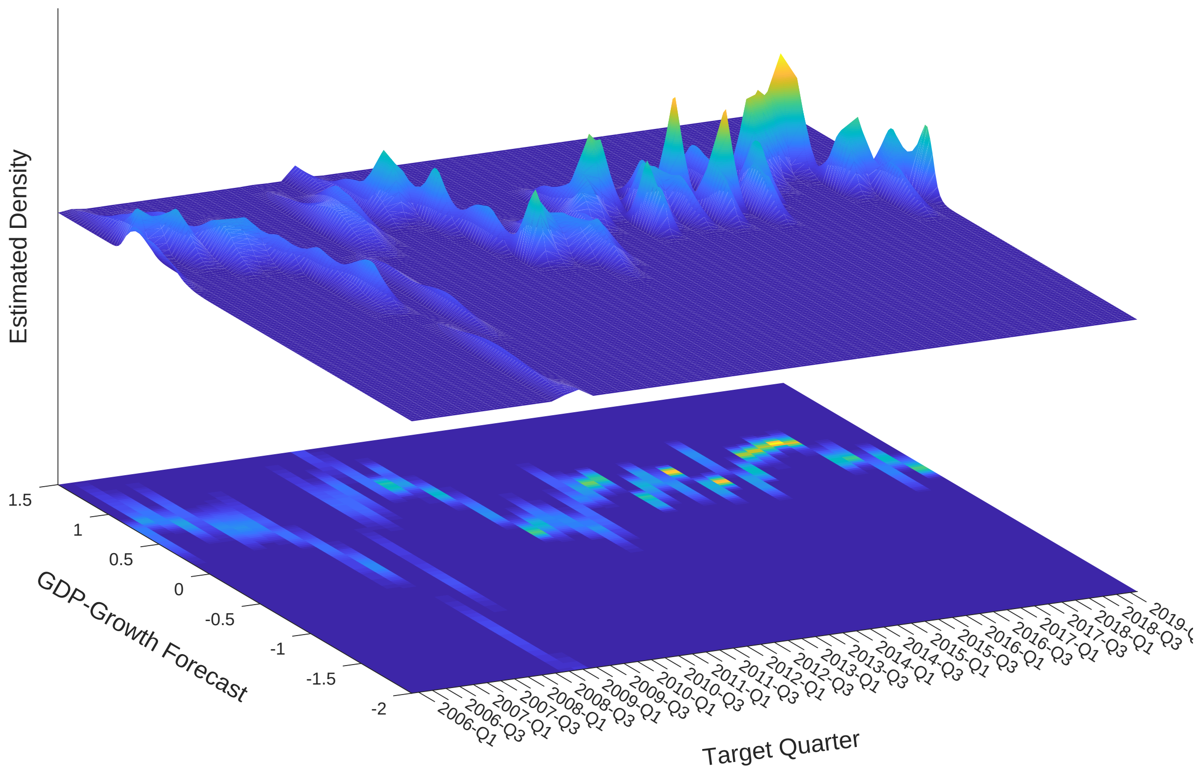
Integrating our suggested model into dynamic model averaging framework enables us to observe the uncertainty around point forecasts. Figure 1 illustrates estimated Kernel densities over the model space for our baseline model for all data vintages at forecast horizon $h = -5$, where the general picture is preserved for other forecast horizons.

Overall, the estimated densities are able to demonstrate forecast uncertainty reasonably well. For instance, in the period surrounding the Great Recession, one can see that the variance of the forecast distribution increases which can be interpreted as increasing forecast disagreement among models. During this period the density is almost entirely flat, indicating that the different models point in vastly different directions. Moreover, a similar pattern emerges to a lesser extend with the aggravation of the euro area sovereign debt crisis during 2011 and 2012. In contrast, toward the end of the evaluation sample, where the German economy was located on a relatively stable growth path, the forecast distributions become heavily concentrated and the models point in very similar directions. On the one hand, this shows that the forecast distributions from our model pick up changes in the economic conditions relatively quickly and react in a way that is reasonable. On the other hand, the changes in forecast disagreement might be useful in determining the risk that surrounds the economy at a given point in time and thus provide insightful information to the forecasters and policy makers.

Despite the increased forecast uncertainty in turbulent times like the Great Recession, our suggested framework is able to generate more reliable forecasts during such periods compared to competing models. Figure 2 displays the cumulative absolute errors (CAE) for $h = -5$ (the general picture is again preserved for other forecast horizons and weighting schemes) in order to assess in which period our model generates forecast performance gains relative to benchmark models.

Overall, Figure 2 demonstrates the superior forecast performance of both TVP-MF-DFM mean and DMA specifications, as also presented in Table 3, because they accumulate less forecast error over time. Given that the slope of the CAE for selected models (with the naive benchmark being the exception) are relatively similar for most of the sample,

Figure 1: Forecast densities and heatmap, $h = -5$

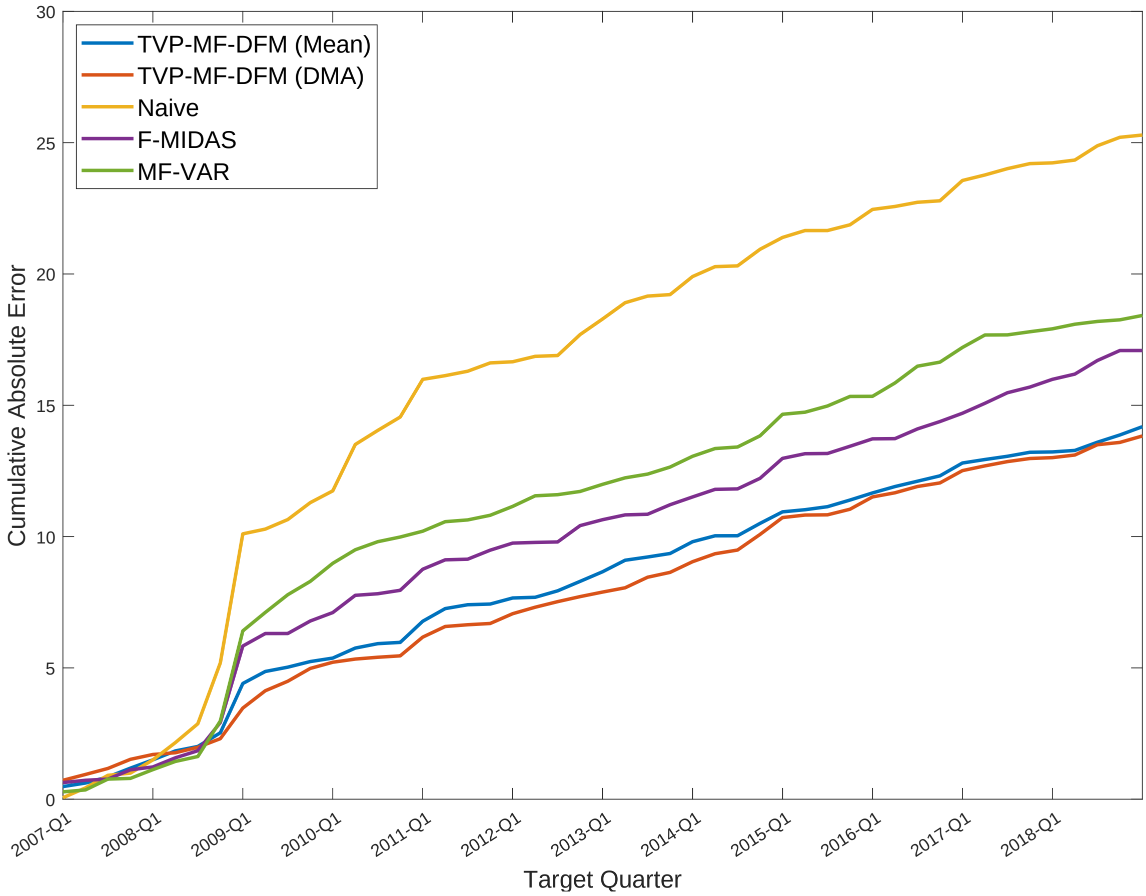


Notes: This figure displays the densities over the forecasts generated by the 65,536 models for $h = -5$ over the estimation sample. The forecast heatmap at the bottom panel gives a two dimensional representation of the estimated densities at the top panel.

it is the financial crisis that makes the difference in forecast performance. The highly flexible TVP-MF-DFM is able to produce more precise forecasts during this period and thus accumulates substantially less error. Rather than providing more precise forecasts where forecast performance is already satisfactory, one advantage of our approach thus seems to be that it provides more precise forecasts during periods of tension and when the economy is on an unstable path or in a transition phase between regimes. This approach can hence be added into the nowcasting toolbox of policy makers as a more reliable nowcasting model in turbulent times.

After having a close look at the forecast performance of the TVP-MF-DFM-DMA we shift our focus on the inner workings of our suggested framework. Since factor models are based on latent and hence unobserved components, they are typically hard to interpret. Usually, it remains unclear why a factor model behaves a certain way or, in the context of forecasting, how forecasts are formed based on the model ingredients. The DMA methodology allows to shed some light onto the inner workings of a factor model or rather forecast combinations thereof. As Koop and Korobilis (2011, 2014) show, one can use the updated individual model probabilities to calculate the expected excess model

Figure 2: Evolution of cumulative forecast errors, $h = -5$



Notes: This figure displays the cumulative absolute errors for selected model variants.

size as well as the importance of the different variables over time:

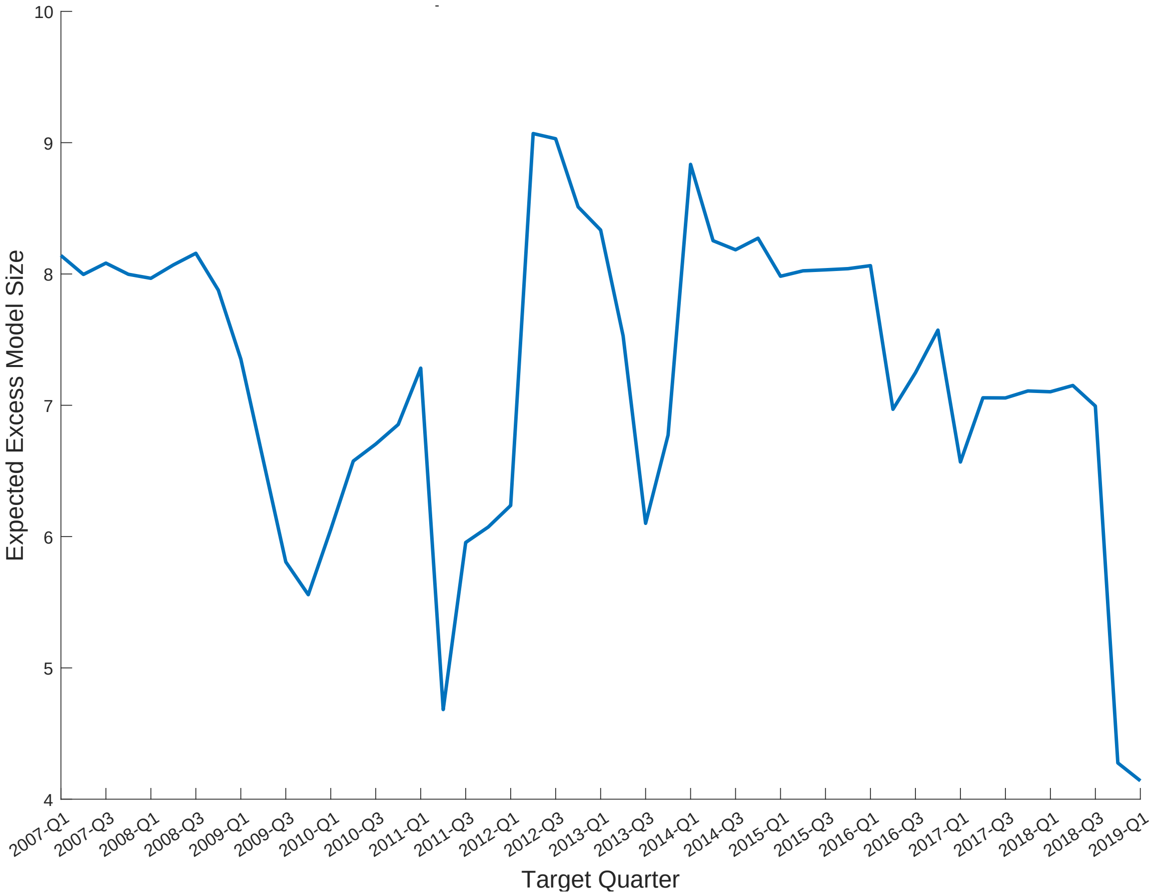
$$E(n_{t,h}) = \left(\sum_{j=1}^{J} \pi_{t|t,j,h} n_j \right) - 4 \tag{18}$$

$$\pi_{t,h}^{x_i} = \sum_{k \subset x_i} \pi_{t|t,j,h} \tag{19}$$

Again we extend on the current framework by allowing both, equations (18) and (19), to depend on the forecast horizon. As we exclude models where no factors are extracted in our empirical exercise, we define the expected model size in excess of the variables that are always included. One can thus either interpret this statistic as the expected excess model size or the number of additional variables DMA/BMA chooses to include when the forecast combination is produced.

Figure 3 displays the expected excess model size for $h = -5$, as given in equation (18). While the expected model consists of around 12 (4 fixed and 8 additional selected) variables on average, there is a considerable fluctuations in the expected number of additional variables over the evaluation period. The excess model size seems to decline in 2008, 2011, 2013 and at the end of our evaluation period. The timing of the first two drops

Figure 3: Expected excess model size, $h = -5$



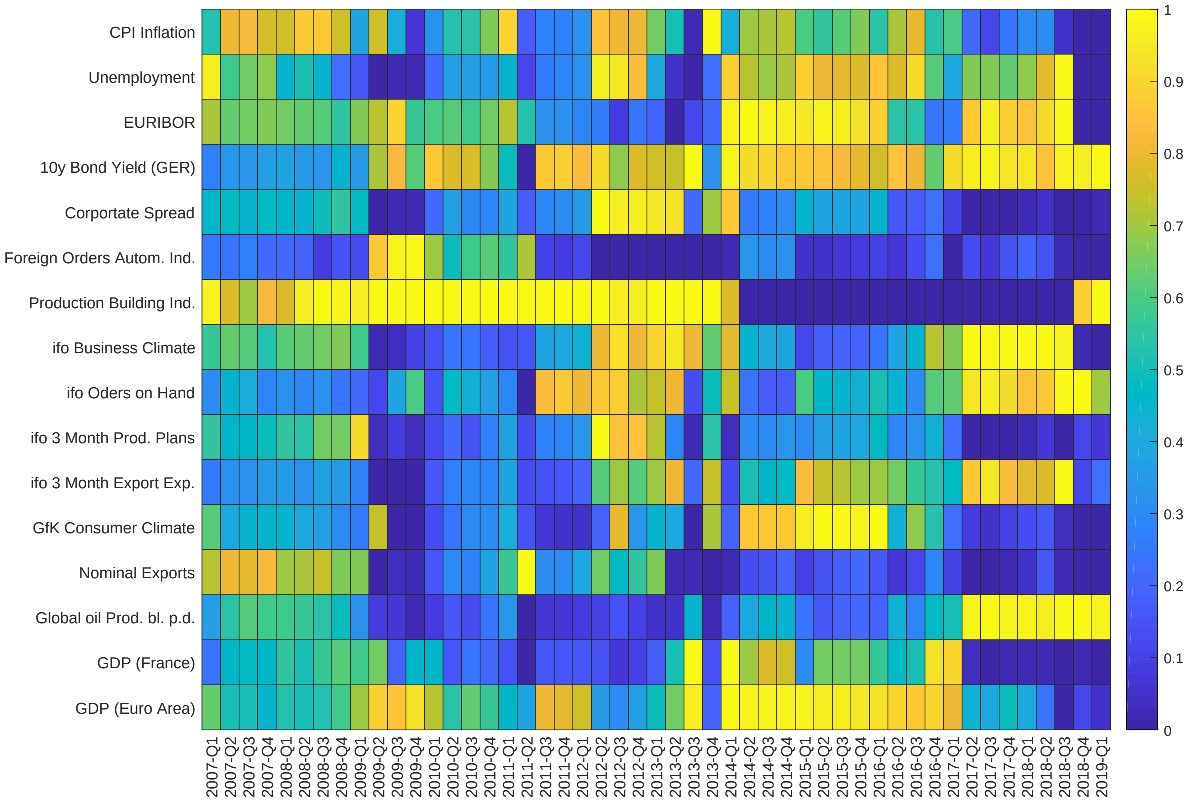
Notes: This figure displays the expected model size in excess of the four variables that are contained in all models. To arrive at the total number of variables one thus has to add four.

in the model size coincides with the Great Recession and euro area sovereign debt crisis, respectively. Moreover, the latter two corresponds to times of a rather decent slowdowns in economic activity. Against this background, in times of high uncertainty parsimonious model specifications appear to be beneficial to the forecast performance, whereas increasing number of indicators seem to improve the forecast accuracy during more tranquil periods. Finally, throughout the evaluation period neither corner solution with either all or no additional variables seems to prevail. This suggests that the additional variables are able to provide information at certain points in time that is not already conveyed by the four variables which are always included in all model specifications through the entire evaluation period.

The fluctuations in expected model size point out that the number of important indicators may change over time, whereas it is not able to identify such variables. Therefore we aim at identifying predictors' probability of being included in the model or loosely speaking importance of the additional variables through time in the final step. This is visualized in the following heatmap.

Figure 4 illustrates the importance of each additional variable over the entire evaluation period. At the beginning of the sample, exports, inflation and the production in the

Figure 4: Indicator heatmap, $h = -5$



Notes: This figure displays the importance of the additional variables in our data set through time. The heatmap is row-scaled, meaning that for each variable the period where it is most (least) important receives the hottest (coldest) color.

building industry seem to contribute relatively more to the DMA forecast than later in the sample. All other indicators receive rather balanced weights of medium magnitude. However, this picture gets disrupted with the onset of the Great Recession. While most variables suddenly loose most of their weights, the EURIBOR, 10y government bond yield, foreign orders in the automotive industry and the euro area GDP experience increases in their relative weights and thus contribution to the DMA forecast. Moreover, indicators, such as inflation, unemployment as well as corporate spread and sentiment indicators appear to gain weight over the course of the euro area sovereign debt crisis. Furthermore, unemployment, EURIBOR, 10y government bond yields, consumer sentiment and the euro area GDP seem to be among most important indicators from 2014 onwards, whereas this picture slightly changes towards the end of our evaluation sample. Since 2017, 10y government bond yields and sentiment indicators, such as business climate and assessment of orders on hand, appear to contribute more to the DMA forecast. Last but not least, global oil production, our proxy for world-wide economic activity, and export expectations gain on weight remarkably during this period.

Finally, a few words of caution are in order, however. Although it is tempting to infer a causal relationship especially when the changes in model weight seem reasonable, one must not do so. First of all, as pointed out, correlation structures between variables can change over time. This holds true for variables within and without the data set. Rather than being important at a certain point in time itself a certain indicator might receive more weight, because it proxies an actually important unobserved predictor. The patterns that emerge might thus be spurious. Second, the weights used to produce these statistics are generated conditional on the forgetting factor γ which governs the dynamics of model switching. An increasingly lower γ implies faster model switching, which is likely to produce a more erratic behavior of the excess model size and a more blurry heatmap. In the extreme case, the simple average, the heatmap will only feature one single color as the weights are identical throughout time and across variables. These figures are thus to be understood as a tool that sheds light on how a certain forecast emerges. They provide the means to observe which and how many variables influenced the forecast the most from the perspective of the model. This in turn allows the forecaster to conduct reality checks assessing whether the model places high weights on indicators that truly seem to be important at the time. Given the usual black-box character of factor models and forecasts generated by their means, this provides additional transparency to the forecaster.

5 Concluding Remarks

In this paper we propose a novel TVP-MF-DFM nowcasting model that can efficiently deal with the characteristics of the real-time data flow as well as parameter instability and time-varying volatility. Moreover, we develop an algorithm optimized for fast estimation that allows us to integrate our TVP-MF-DFMs into a dynamic model averaging framework. This enables us to generate forecasts based on a large model space which is advisable when faced with model uncertainty.

We put our model to the test in a pseudo real-time recursive out-of-sample forecasting exercise. Our results reveal that it can realize performance gains relative to both a naive benchmark and more competitive models of similar nature in forecasting German GDP growth. While our proposed model performs similar to the competitive benchmarks during rather tranquil sample periods, it improves upon the nowcasting accuracy of the benchmark models especially during the financial crisis, where our model produces more precise forecasts. Overall, the proposed model specifications manage to improve the forecast accuracy of the naive benchmark by up to 61%. Compared to the competitive benchmarks, forecast performance improves by up to 10% on average and by up to 40% in the most favourable case, where the results generally depend on the forecast performance measure, the model averaging method, and the forecast horizon. Furthermore, we find that performance based weighting schemes suffer from update lag as the forecast horizon and publication lag of the target variable increases, which might explain why they seem to improve upon simple weighting schemes only for short horizons. Finally, we show how the DMA methodology can be used to assess which variables are most influential for a given forecast, which provides additional transparency to the forecaster. Providing considerable improvements in forecast accuracy and additional transparency our suggested framework is a useful complement to the forecasting toolbox of policy makers.

This paper also opens up new avenues for further research in the nowcasting and the model averaging literature. Future work could, e.g., investigate the potential of allowing not only the parameters and models to change over time, but also the decay parameters and forgetting factors to be dynamic. Moreover, one could allow for different decay rates for each variable and different degrees of model change for each forecast horizon. Such extensions of our framework, however, are left for further research.

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Appendix A

A.1 Kalman filter

The objective is to update what we know about the system each time a new observation arrives to produce estimates of the latent state variables. One can derive the Kalman filter's update and prediction equations, starting by assuming that all variables involved are normally distributed. By the properties of the normal distribution, conditional distributions of one subset of variables conditional on another remain normally distributed. Thus, the expressions for the respective conditional means and conditional variances, which are known from regression theory, apply (for detailed proof, see e.g. Blake and Mumtaz, 2017; Durbin and Koopman, 2012). Proceeding from here, one can use these results and derive the expressions for the conditional moments of the state variables. Conditional on the observation equation, (7a), and the factor state equation, (7b), the Kalman filter equations are then given by,

$$s_{t|t-1} = A_t s_{t-1|t-1} \tag{20a}$$
$$\Sigma^f_{t|t-1} = A_t \Sigma^f_{t-1|t-1} A'_t + Q_t \tag{20b}$$

$$\xi_t = x_t - H_t s_{t|t-1} \tag{20c}$$
$$F_t = H_t \Sigma^f_{t|t-1} H'_t + V_t \tag{20d}$$
$$s_{t|t} = s_{t|t-1} + \Sigma^f_{t|t-1} H'_t F_t^{-1} \xi_t \tag{20e}$$
$$\Sigma^f_{t|t} = \Sigma^f_{t|t-1} - \Sigma^f_{t|t-1} H'_t F_t^{-1} H_t \Sigma^f_{t|t-1} \tag{20f}$$

where $s_t|Data_{1:t-1} \sim N(s_{t|t-1}, \Sigma^f_{t|t-1})$ and $s_t|Data_{1:t} \sim N(s_{t|t}, \Sigma^f_{t|t})$. Equations (20a) and (20b) are known as the prediction equations, whereas equations (20e) and (20f) represent the update equations. From a Bayesian point of view, the Kalman filter provides vectors and matrices of quasi-posterior means and variances, respectively, which feature minimum variance linear unbiased interpretations. From the standpoint of classic inference, the Kalman filter produces minimum variance linear unbiased estimates of the state variables. Importantly, this holds true whether or not the variables involved are normally distributed (see Durbin and Koopman, 2012). Although one usually assumes normality, the Kalman filter equations thus remain valid under more general circumstances. Finally, the equations for the fixed interval smoother, following Rauch et al. (1965), are given by

$$C^f_t = \Sigma^f_{t|t} A'_{t|T} \left(\Sigma^f_{t+1|t}\right)^{-1} \tag{21a}$$

$$s_{t|T} = s_{t|t} + C^f_t (s_{t+1|T} - s_{t+1|t}) \tag{21b}$$
$$\Sigma^f_{t|T} = \Sigma^f_{t|t} + C^f_t (\Sigma^f_{t+1|T} - \Sigma^f_{t+1|t})(C^f_t)' \tag{21c}$$

While the Kalman filter considers information up to time t and proceeds forward through time, the smoother considers the entire data series and proceeds backwards through time. From the perspective of a Kalman filter iteration at time t, the smoother thus also takes into account information that lies in the future and adjusts the Kalman filter output appropriately.

A.2 Grid Search

As pointed out in the main body of the paper, we use a simple grid search to guide our choice of parameters. Since κ_1^M and κ_2 as well as κ_3 and κ_4 are defined at the same frequency and govern the degree of time-variation of similar components, we restrict them to the same value. κ_1^Q is responsible for the parameter change of the residual variances for the quarterly variables and allowed to take its own value. Thus our grid is three dimensional, which eases the computational burden.

$$\begin{aligned}
\kappa_1^M, \kappa_2 &\in \begin{bmatrix} 0.60 & 0.65 & 0.70 & 0.75 & 0.80 & 0.85 & 0.90 & 0.95 & 0.99 \end{bmatrix} \\
\kappa_1^Q &\in \begin{bmatrix} 0.60 & 0.65 & 0.70 & 0.75 & 0.80 & 0.85 & 0.90 & 0.95 & 0.99 \end{bmatrix} \\
\kappa_3, \kappa_4 &\in \begin{bmatrix} 0.90 & 0.91 & 0.92 & 0.93 & 0.94 & 0.95 & 0.96 & 0.97 & 0.98 & 0.99 \end{bmatrix}
\end{aligned}$$

Given the high computational demand of our approach, we evaluate the grid for the model containing all predictors instead of the entire model space. This should produce a parameter specification that is somewhat appropriate for all variables instead of only a selected few.

As our parameter specification we then adopt the grid point that minimizes the average mean absolute deviation (MAD) over the nowcast and backcast horizon. Generally, to fit the individual forecaster's needs one could also choose to minimize the average MAD over all forecast horizons or any combination thereof. This allows to design models that are specialized on e.g. forecasting, nowcasting and backcasting. Forecasts over the entire set of forecast horizons then arise from different models that are estimated independently. In case of our model, however, this would triple the computational demand, which is why we abstract from this approach.

A.3 Sensitivity Analysis

In addition to our baseline modelling framework we also evaluate various model specifications in order to check the sensitivity of forecast performance to different factor structures and decay parameters in dynamic model averaging. While the overall forecast performance deteriorates with decreasing number of factors, the results seem to be less sensitive to the forgetting factor which governs the rate of model switching. The results for various model specifications are presented in Tables A.1 − A.11.

Table A.1: Relative forecast performance, 3 Factors, $\gamma = 0.8$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.00	0.99	0.96	0.97	0.87**	0.86**	0.88*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.64**	0.59**	0.59**	0.56***
TVP-MF-DFM Median	1.00	0.98	0.96	0.98	0.87**	0.86**	0.89*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.65**	0.59**	0.59**	0.57***
TVP-MF-DFM DMS	1.03	1.03	1.01	1.03	0.95	0.93	0.95	0.94	0.90	0.88*	0.85*	0.84	0.81*	0.67**	0.73**	0.68*	0.59**	0.58**
TVP-MF-DFM DMA	0.99	1.01	0.98	1.03	0.89*	0.90*	0.89*	0.88*	0.86**	0.87*	0.88	0.84*	0.82*	0.65**	0.67**	0.59**	0.59**	0.55**
TVP-MF-DFM BMA	1.02	1.04	1.00	1.05	0.91	0.91	0.91*	0.90*	0.89	0.87*	0.87*	0.80**	0.77**	0.63**	0.64**	0.57**	0.59**	0.54**
MF-DFM Mean	0.92*	0.92**	0.90*	0.92	0.87**	0.84***	0.98	0.94	0.91	0.96	1.01	0.90	0.86	0.64**	0.63**	0.58**	0.59**	0.54***
MF-DFM Median	0.92*	0.92**	0.89*	0.92	0.87**	0.84***	1.00	0.94	0.92	0.96	1.00	0.90	0.86	0.64**	0.63**	0.59**	0.59**	0.54**
MF-DFM DMS	0.98	1.02	0.95	0.95	1.00	1.02	0.86**	1.06	0.93	1.16	1.19	0.94	1.15	0.70*	0.68**	0.59**	0.63*	0.50**
MF-DFM DMA	0.95	0.98	0.90*	0.90**	0.95	0.93	0.93	1.06	0.90	1.15	1.13	0.94	1.04	0.67**	0.65**	0.59**	0.61**	0.51**
MF-DFM BMA	0.94	0.90**	0.87**	0.93	0.92	0.94	0.94	1.06	0.98	1.19	1.16	0.92	1.02	0.67**	0.64**	0.69*	0.62*	0.54**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.51*	0.45*	0.46*	0.44*
TVP-MF-DFM Median	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.50*	0.45*	0.45*	0.43*
TVP-MF-DFM DMS	0.97	1.00	1.01	1.04	0.96	0.93	0.86	0.84	0.80	0.76	0.77*	0.66	0.78*	0.52*	0.60*	0.47*	0.43*	0.40*
TVP-MF-DFM DMA	0.95	0.98	0.96	1.01	0.90*	0.91	0.78*	0.80*	0.78*	0.77	0.79*	0.71	0.74*	0.51*	0.53*	0.41*	0.41*	0.39*
TVP-MF-DFM BMA	0.96	1.00	0.99	1.04	0.92*	0.95	0.81	0.84*	0.80*	0.76	0.76*	0.69*	0.71*	0.47*	0.48*	0.40*	0.41*	0.39*
MF-DFM Mean	0.93**	0.94**	0.92**	0.95*	0.90**	0.89**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.67	0.47*	0.46*	0.42*	0.43*	0.42*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.90**	0.88**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.68	0.47*	0.47*	0.43*	0.43*	0.42*
MF-DFM DMS	1.01	0.97	0.92	0.91	1.02	0.95	0.82*	1.05	0.76	0.95	0.88	0.78	0.97	0.49*	0.48*	0.40*	0.42*	0.36*
MF-DFM DMA	1.00	0.95	0.91*	0.93*	1.00	0.94	0.84	1.04	0.76	0.95	0.86	0.78	0.91	0.48*	0.48*	0.39*	0.40*	0.37*
MF-DFM BMA	1.00	0.89*	0.88*	1.02	0.94*	0.96	0.86	1.05	0.87	0.96	0.86	0.78	0.83	0.47*	0.47*	0.44*	0.40*	0.39*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: The forecast horizon h is given as the distance to the end of the reference quarter measured in weeks. Negative values indicate backcasts. The benchmark is the in-sample mean. ***, ** and * indicate one-sided Diebold-Mariano p-values that are smaller or equal to 0.01, 0.05 and 0.1, respectively.

Table A.2: Relative forecast performance, 3 Factors, $\gamma = 0.7$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.00	0.99	0.96	0.97	0.87**	0.86**	0.88*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.64**	0.59**	0.59**	0.56***
TVP-MF-DFM Median	1.00	0.98	0.96	0.98	0.87**	0.86**	0.89*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.65**	0.59**	0.59**	0.57***
TVP-MF-DFM DMS	1.02	0.98	1.00	1.04	0.98	0.93	0.91	0.93	0.91	0.89	0.84*	0.87	0.82*	0.70**	0.76*	0.68*	0.64**	0.62**
TVP-MF-DFM DMA	1.00	1.01	0.98	1.01	0.88**	0.87**	0.89*	0.87**	0.85**	0.87*	0.89	0.84*	0.83*	0.65**	0.68**	0.59**	0.58**	0.55**
TVP-MF-DFM BMA	1.02	1.04	1.00	1.05	0.91	0.91	0.91*	0.90*	0.89	0.87*	0.87*	0.80**	0.77**	0.63**	0.64**	0.57**	0.59**	0.54**
MF-DFM Mean	0.92*	0.92**	0.90*	0.92	0.87**	0.84***	0.98	0.94	0.91	0.96	1.01	0.90	0.86	0.64**	0.63**	0.58**	0.59**	0.54***
MF-DFM Median	0.92*	0.92**	0.89*	0.92	0.87**	0.84***	1.00	0.94	0.92	0.96	1.00	0.90	0.86	0.64**	0.63**	0.59**	0.59**	0.54**
MF-DFM DMS	0.97	1.01	0.96	0.96	0.99	0.98	0.86**	1.04	0.95	1.21	1.15	0.98	1.11	0.69*	0.68**	0.64*	0.65*	0.52**
MF-DFM DMA	0.93*	0.98	0.90*	0.89**	0.93	0.91	0.92	1.03	0.92	1.17	1.15	0.94	1.06	0.65**	0.65**	0.61**	0.62**	0.53**
MF-DFM BMA	0.94	0.90**	0.87**	0.93	0.92	0.94	0.94	1.06	0.98	1.19	1.16	0.92	1.02	0.67**	0.64**	0.69*	0.62*	0.54**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.51*	0.45*	0.46*	0.44*
TVP-MF-DFM Median	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.50*	0.45*	0.45*	0.43*
TVP-MF-DFM DMS	0.97	0.97	1.01	1.03	0.95	0.90	0.80	0.82	0.81	0.79	0.76	0.69	0.79*	0.54*	0.61*	0.46*	0.46*	0.42*
TVP-MF-DFM DMA	0.95	0.99	0.97	1.00	0.89*	0.89*	0.78*	0.80*	0.77*	0.78	0.80*	0.72	0.73*	0.52*	0.54*	0.41*	0.41*	0.39*
TVP-MF-DFM BMA	0.96	1.00	0.99	1.04	0.92*	0.95	0.81	0.84*	0.80*	0.76	0.76*	0.69*	0.71*	0.47*	0.48*	0.40*	0.41*	0.39*
MF-DFM Mean	0.93**	0.94**	0.92**	0.95*	0.90**	0.89**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.67	0.47*	0.46*	0.42*	0.43*	0.42*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.90**	0.88**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.68	0.47*	0.47*	0.43*	0.43*	0.42*
MF-DFM DMS	1.00	0.97	0.94	0.92	0.96	0.94	0.82*	0.94	0.81	0.96	0.87	0.79	0.93	0.50	0.51*	0.43*	0.44	0.37*
MF-DFM DMA	0.99	0.95	0.92*	0.92*	0.93	0.92	0.83	0.90	0.75	0.95	0.86	0.78	0.89	0.47*	0.49*	0.42*	0.42*	0.38*
MF-DFM BMA	1.00	0.89*	0.88*	1.02	0.94*	0.96	0.86	1.05	0.87	0.96	0.86	0.78	0.83	0.47*	0.47*	0.44*	0.40*	0.39*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.3: Relative forecast performance, 3 Factors, $\gamma = 0.6$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.00	0.99	0.96	0.97	0.87**	0.86**	0.88*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.64**	0.59**	0.59**	0.56***
TVP-MF-DFM Median	1.00	0.98	0.96	0.98	0.87**	0.86**	0.89*	0.86**	0.83**	0.84**	0.87*	0.79**	0.79**	0.64**	0.65**	0.59**	0.59**	0.57***
TVP-MF-DFM DMS	1.03	1.03	0.96	1.05	1.00	0.92	0.91	0.91	0.88*	0.94	0.86*	0.89	0.89	0.66**	0.77*	0.70*	0.63**	0.63**
TVP-MF-DFM DMA	1.00	1.02	0.98	1.00	0.87**	0.87**	0.90*	0.87**	0.85**	0.87*	0.89	0.84*	0.84*	0.66**	0.68**	0.58**	0.58**	0.55**
TVP-MF-DFM BMA	1.02	1.04	1.00	1.05	0.91	0.91	0.91*	0.90*	0.89	0.87*	0.87*	0.80**	0.77**	0.63**	0.64**	0.57**	0.59**	0.54**
MF-DFM Mean	0.92*	0.92**	0.90*	0.92	0.87**	0.84***	0.98	0.94	0.91	0.96	1.01	0.90	0.86	0.64**	0.63**	0.58**	0.59**	0.54***
MF-DFM Median	0.92*	0.92**	0.89*	0.92	0.87**	0.84***	1.00	0.94	0.92	0.96	1.00	0.90	0.86	0.64**	0.63**	0.59**	0.59**	0.54**
MF-DFM DMS	0.98	1.02	0.92**	1.00	0.99	0.99	0.84**	1.09	0.94	1.27	1.21	0.94	1.18	0.72*	0.72*	0.66*	0.64*	0.54**
MF-DFM DMA	0.94	0.98	0.89**	0.93	0.93	0.91	0.91	1.02	0.90	1.18	1.14	0.94	1.07	0.66*	0.66**	0.61**	0.61**	0.54**
MF-DFM BMA	0.94	0.90**	0.87**	0.93	0.92	0.94	0.94	1.06	0.98	1.19	1.16	0.92	1.02	0.67**	0.64**	0.69*	0.62*	0.54**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.51*	0.45*	0.46*	0.44*
TVP-MF-DFM Median	0.96	0.97	0.96	1.00	0.88*	0.88*	0.78*	0.79*	0.75*	0.75	0.78*	0.70*	0.69*	0.51*	0.50*	0.45*	0.45*	0.43*
TVP-MF-DFM DMS	0.96	1.02	0.99	1.03	0.99	0.89	0.83*	0.82	0.77	0.81	0.82	0.75	0.81*	0.52*	0.63*	0.48	0.45*	0.42*
TVP-MF-DFM DMA	0.96	1.00	0.97	1.00	0.88*	0.88*	0.78*	0.80*	0.77*	0.78	0.80*	0.72	0.73*	0.52*	0.54*	0.41*	0.41*	0.39*
TVP-MF-DFM BMA	0.96	1.00	0.99	1.04	0.92*	0.95	0.81	0.84*	0.80*	0.76	0.76*	0.69*	0.71*	0.47*	0.48*	0.40*	0.41*	0.39*
MF-DFM Mean	0.93**	0.94**	0.92**	0.95*	0.90**	0.89**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.67	0.47*	0.46*	0.42*	0.43*	0.42*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.90**	0.88**	0.87	0.84*	0.81*	0.81	0.83	0.73	0.68	0.47*	0.47*	0.43*	0.43*	0.42*
MF-DFM DMS	1.00	0.97	0.91*	0.97	0.96	0.95	0.82*	0.95	0.80	0.99	0.92	0.78	0.96	0.52	0.54	0.45	0.43*	0.39*
MF-DFM DMA	0.99	0.96	0.91*	0.95	0.93	0.92	0.84	0.91	0.74	0.95	0.83	0.77	0.90	0.47*	0.50*	0.42*	0.41*	0.40*
MF-DFM BMA	1.00	0.89*	0.88*	1.02	0.94*	0.96	0.86	1.05	0.87	0.96	0.86	0.78	0.83	0.47*	0.47*	0.44*	0.40*	0.39*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.4: Relative forecast performance, 2 Factors, $\gamma = 0.9$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.01	1.01	0.97	0.97	0.87**	0.84***	0.93	0.84**	0.84*	0.84**	0.86*	0.79**	0.80*	0.71**	0.70**	0.68**	0.69**	0.67**
TVP-MF-DFM Median	1.01	1.00	0.97	0.96	0.87**	0.84***	0.93	0.85**	0.84*	0.84**	0.86*	0.79**	0.80*	0.72**	0.71**	0.70**	0.71**	0.68**
TVP-MF-DFM DMS	1.00	0.97	0.92	0.95	0.96	0.85***	0.87*	0.83**	0.87	0.92	0.89	0.78**	0.95	0.80	0.85	0.69*	0.68*	0.68*
TVP-MF-DFM DMA	0.98	0.99	0.93	0.91*	0.87**	0.85***	0.90	0.86**	0.89	0.90	0.86*	0.79**	0.87	0.73**	0.74**	0.66*	0.66*	0.67**
TVP-MF-DFM BMA	1.00	0.97	0.98	0.93*	0.88**	0.83***	0.94	0.83**	0.86*	0.92	0.85*	0.77**	0.86	0.76*	0.75*	0.64*	0.67*	0.66**
MF-DFM Mean	0.91*	0.92*	0.90*	0.92	0.89**	0.85***	0.98	0.92	0.90	0.92	0.94	0.85	0.83*	0.70**	0.68**	0.68**	0.69**	0.66**
MF-DFM Median	0.91*	0.92**	0.89*	0.92	0.89**	0.85***	0.99	0.92	0.90	0.92	0.94	0.85	0.83*	0.71**	0.70**	0.70**	0.70**	0.67**
MF-DFM DMS	0.95	1.09	1.05	0.98	1.07	1.04	1.00	1.00	0.99	1.03	1.02	0.86	0.97	0.61**	0.61**	0.61**	0.58**	0.63***
MF-DFM DMA	0.93	1.00	0.94	0.91*	1.00	0.94	1.01	0.99	0.98	1.00	1.05	0.83*	0.99	0.60**	0.60**	0.59**	0.59**	0.63***
MF-DFM BMA	0.93	0.97	0.93	0.91*	0.99	0.92	1.02	0.98	1.07	1.02	1.01	0.84*	1.06	0.66*	0.61**	0.56**	0.56**	0.60***
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.97	0.98	0.97	0.97	0.88*	0.83*	0.78	0.75*	0.71*	0.76*	0.76*	0.72*	0.68*	0.60*	0.59*	0.58*	0.58*	0.56*
TVP-MF-DFM Median	0.97	0.98	0.97	0.97	0.87*	0.83*	0.77	0.75*	0.71	0.76*	0.75*	0.71*	0.67*	0.61*	0.60*	0.59*	0.59*	0.56*
TVP-MF-DFM DMS	0.99	0.97	0.93	0.96	0.95	0.86*	0.73	0.76*	0.70	0.78	0.78*	0.72*	0.90*	0.59	0.66	0.48	0.47	0.47*
TVP-MF-DFM DMA	0.95	0.97	0.93	0.94**	0.85*	0.85*	0.75	0.76	0.75	0.78	0.73*	0.71*	0.78*	0.58*	0.58*	0.45*	0.46*	0.49*
TVP-MF-DFM BMA	0.96	0.96	0.95	0.95**	0.86*	0.84**	0.75	0.73	0.72	0.77	0.74*	0.70*	0.74*	0.57*	0.60*	0.44*	0.45*	0.48*
MF-DFM Mean	0.93**	0.94**	0.92*	0.95*	0.90**	0.88**	0.84	0.80	0.74	0.76	0.77	0.72*	0.68*	0.58*	0.57*	0.56*	0.56*	0.55*
MF-DFM Median	0.93**	0.94**	0.92*	0.95*	0.89**	0.87**	0.84	0.81	0.73	0.76	0.78	0.72*	0.68*	0.59*	0.58*	0.58*	0.58*	0.56*
MF-DFM DMS	0.98	1.01	0.99	0.98	1.01	0.97	0.92	0.86	0.76	0.80	0.76	0.74	0.82	0.45*	0.44*	0.42*	0.41*	0.54*
MF-DFM DMA	0.98	0.98	0.95	0.97	0.99	0.93	0.91	0.86	0.74	0.80	0.77	0.71	0.83	0.44*	0.44*	0.41*	0.41*	0.53*
MF-DFM BMA	1.01	0.95	0.95	0.97	0.98	0.91*	0.91	0.82	0.86	0.80	0.79	0.72	0.92	0.47*	0.44*	0.41*	0.41*	0.48*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.5: Relative forecast performance, 2 Factors, $\gamma = 0.8$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.01	1.01	0.97	0.97	0.87**	0.84***	0.93	0.84**	0.84*	0.84**	0.86*	0.79**	0.80*	0.71**	0.70**	0.68**	0.69**	0.67**
TVP-MF-DFM Median	1.01	1.00	0.97	0.96	0.87**	0.84***	0.93	0.85**	0.84*	0.84**	0.86*	0.79**	0.80*	0.72**	0.71**	0.70**	0.71**	0.68**
TVP-MF-DFM DMS	0.97	1.00	0.92	0.95	0.94	0.86***	0.91	0.87**	0.98	0.94	0.88	0.78**	0.94	0.78*	0.83	0.67*	0.67*	0.68**
TVP-MF-DFM DMA	0.97	1.00	0.94	0.92*	0.89**	0.83***	0.91	0.88*	0.90	0.91	0.88	0.80**	0.88	0.73**	0.74**	0.67*	0.67**	0.68**
TVP-MF-DFM BMA	1.00	0.97	0.98	0.93*	0.88**	0.83***	0.94	0.83**	0.86*	0.92	0.85*	0.77**	0.86	0.76*	0.75*	0.64*	0.67*	0.66**
MF-DFM Mean	0.91*	0.92*	0.90*	0.92	0.89**	0.85***	0.98	0.92	0.90	0.92	0.94	0.85	0.83*	0.70**	0.68**	0.68**	0.69**	0.66**
MF-DFM Median	0.91*	0.92**	0.89*	0.92	0.89**	0.85***	0.99	0.92	0.90	0.92	0.94	0.85	0.83*	0.71**	0.70**	0.70**	0.70**	0.67**
MF-DFM DMS	0.96	1.08	1.04	0.98	1.05	1.04	0.98	0.99	0.98	1.05	1.10	0.88	1.02	0.61**	0.65**	0.57**	0.59**	0.64**
MF-DFM DMA	0.93	0.99	0.93	0.91*	0.98	0.93	0.99	0.99	0.95	1.03	1.06	0.86	1.01	0.60**	0.62**	0.58**	0.60**	0.65**
MF-DFM BMA	0.93	0.97	0.93	0.91*	0.99	0.92	1.02	0.98	1.07	1.02	1.01	0.84*	1.06	0.66*	0.61**	0.56**	0.56**	0.60***
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.97	0.98	0.97	0.97	0.88*	0.83*	0.78	0.75*	0.71*	0.76*	0.76*	0.72*	0.68*	0.60*	0.59*	0.58*	0.58*	0.56*
TVP-MF-DFM Median	0.97	0.98	0.97	0.97	0.87*	0.83*	0.77	0.75*	0.71	0.76*	0.75*	0.71*	0.67*	0.61*	0.60*	0.59*	0.59*	0.56*
TVP-MF-DFM DMS	0.95	0.98	0.92	0.97	0.93	0.85*	0.76	0.82*	0.84	0.83	0.75	0.70*	0.89**	0.64*	0.65	0.47	0.47	0.49*
TVP-MF-DFM DMA	0.95	0.98	0.94	0.94**	0.86*	0.83*	0.78	0.77	0.76	0.80	0.74	0.72*	0.80*	0.60*	0.59*	0.47*	0.47*	0.51*
TVP-MF-DFM BMA	0.96	0.96	0.95	0.95**	0.86*	0.84**	0.75	0.73	0.72	0.77	0.74*	0.70*	0.74*	0.57*	0.60*	0.44*	0.45*	0.48*
MF-DFM Mean	0.93**	0.94**	0.92*	0.95*	0.90**	0.88**	0.84	0.80	0.74	0.76	0.77	0.72*	0.68*	0.58*	0.57*	0.56*	0.56*	0.55*
MF-DFM Median	0.93**	0.94**	0.92*	0.95*	0.89**	0.87**	0.84	0.81	0.73	0.76	0.78	0.72*	0.68*	0.59*	0.58*	0.58*	0.58*	0.56*
MF-DFM DMS	1.02	1.00	0.99	0.98	0.98	0.98	0.87	0.88	0.74	0.81	0.82	0.70	0.92	0.45*	0.46*	0.41*	0.42*	0.49*
MF-DFM DMA	1.01	0.97	0.95	0.97	0.97	0.93	0.87	0.86	0.73	0.82	0.80	0.70	0.89	0.44*	0.44*	0.41*	0.41*	0.49*
MF-DFM BMA	1.01	0.95	0.95	0.97	0.98	0.91*	0.91	0.82	0.86	0.80	0.79	0.72	0.92	0.47*	0.44*	0.41*	0.41*	0.48*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.6: Relative forecast performance, 2 Factors, $\gamma = 0.7$

h=	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	-1	-3	-5
Relative MAD																		
TVP-MF-DFM Mean	1.01	1.01	0.97	0.97	0.87**	0.84***	0.93	0.84**	0.84*	0.84**	0.86*	0.79**	0.80*	0.71**	0.70**	0.68**	0.69**	0.67**
TVP-MF-DFM Median	1.01	1.00	0.97	0.96	0.87**	0.84***	0.93	0.85**	0.84*	0.84**	0.86*	0.79**	0.80*	0.72**	0.71**	0.70**	0.71**	0.68**
TVP-MF-DFM DMS	1.01	1.06	0.97	0.99	0.93	0.89**	0.90	0.89*	0.93	0.95	0.89	0.77**	0.95	0.81*	0.80*	0.67*	0.69*	0.71*
TVP-MF-DFM DMA	0.98	1.01	0.98	0.93*	0.88**	0.84***	0.92	0.89*	0.90	0.92	0.89	0.81*	0.88	0.75**	0.74**	0.68*	0.68**	0.69**
TVP-MF-DFM BMA	1.00	0.97	0.98	0.93*	0.88**	0.83***	0.94	0.83**	0.86*	0.92	0.85*	0.77**	0.86	0.76*	0.75*	0.64*	0.67*	0.66**
MF-DFM Mean	0.91*	0.92*	0.90*	0.92	0.89**	0.85***	0.98	0.92	0.90	0.92	0.94	0.85	0.83*	0.70**	0.68**	0.68**	0.69**	0.66**
MF-DFM Median	0.91*	0.92**	0.89*	0.92	0.89**	0.85***	0.99	0.92	0.90	0.92	0.94	0.85	0.83*	0.71**	0.70**	0.70**	0.70**	0.67**
MF-DFM DMS	0.95	1.08	1.04	0.98	1.05	1.02	0.97	0.98	0.99	1.06	1.12	0.92	1.06	0.68*	0.68*	0.64**	0.66**	0.66**
MF-DFM DMA	0.93*	0.99	0.93	0.91*	0.99	0.94	0.99	1.00	0.96	1.03	1.11	0.89	0.98	0.63**	0.65**	0.63**	0.62**	0.66**
MF-DFM BMA	0.93	0.97	0.93	0.91*	0.99	0.92	1.02	0.98	1.07	1.02	1.01	0.84*	1.06	0.66*	0.61**	0.56**	0.56**	0.60***
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.97	0.98	0.97	0.97	0.88*	0.83*	0.78	0.75*	0.71*	0.76*	0.76*	0.72*	0.68*	0.60*	0.59*	0.58*	0.58*	0.56*
TVP-MF-DFM Median	0.97	0.98	0.97	0.97	0.87*	0.83*	0.77	0.75*	0.71	0.76*	0.75*	0.71*	0.67*	0.61*	0.60*	0.59*	0.59*	0.56*
TVP-MF-DFM DMS	0.97	1.01	0.94	0.99	0.92*	0.89*	0.76	0.80	0.77	0.86	0.75	0.71*	0.90*	0.65	0.64*	0.47	0.48	0.50
TVP-MF-DFM DMA	0.95	0.98	0.96	0.95**	0.88**	0.83*	0.80	0.78	0.77	0.82	0.76	0.72*	0.79*	0.61*	0.59*	0.49*	0.49*	0.53*
TVP-MF-DFM BMA	0.96	0.96	0.95	0.95**	0.86*	0.84**	0.75	0.73	0.72	0.77	0.74*	0.70*	0.74*	0.57*	0.60*	0.44*	0.45*	0.48*
MF-DFM Mean	0.93**	0.94**	0.92*	0.95*	0.90**	0.88**	0.84	0.80	0.74	0.76	0.77	0.72*	0.68*	0.58*	0.57*	0.56*	0.56*	0.55*
MF-DFM Median	0.93**	0.94**	0.92*	0.95*	0.89**	0.87**	0.84	0.81	0.73	0.76	0.78	0.72*	0.68*	0.59*	0.58*	0.58*	0.58*	0.56*
MF-DFM DMS	1.02	1.00	0.98	0.99	0.98	0.98	0.90	0.85	0.74	0.82	0.86	0.71	1.03	0.48*	0.47*	0.43*	0.45*	0.50*
MF-DFM DMA	1.00	0.97	0.95	0.97	0.98	0.94	0.88	0.87	0.74	0.84	0.88	0.72	0.91	0.45*	0.45*	0.42*	0.42*	0.51*
MF-DFM BMA	1.01	0.95	0.95	0.97	0.98	0.91*	0.91	0.82	0.86	0.80	0.79	0.72	0.92	0.47*	0.44*	0.41*	0.41*	0.48*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.7: Relative forecast performance, 2 Factors, $\gamma = 0.6$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	1.01	1.01	0.97	0.97	0.87**	0.84***	0.93	0.84**	0.84*	0.84**	0.86*	0.79**	0.80*	0.71**	0.70**	0.68**	0.69**	0.67**
TVP-MF-DFM Median	1.01	1.00	0.97	0.96	0.87**	0.84***	0.93	0.85**	0.84*	0.84**	0.86*	0.79**	0.80*	0.72**	0.71**	0.70**	0.71**	0.68**
TVP-MF-DFM DMS	1.02	1.05	1.03	0.97	0.95	0.98	0.94	0.86*	0.93	0.94	0.92	0.83*	0.94	0.83	0.83	0.68*	0.72*	0.72*
TVP-MF-DFM DMA	0.98	1.02	1.00	0.95*	0.88**	0.85***	0.92	0.89*	0.89	0.92	0.90	0.81*	0.87	0.74**	0.74**	0.68**	0.68**	0.69**
TVP-MF-DFM BMA	1.00	0.97	0.98	0.93*	0.88**	0.83***	0.94	0.83**	0.86*	0.92	0.85*	0.77**	0.86	0.76*	0.75*	0.64*	0.67*	0.66**
MF-DFM Mean	0.91*	0.92*	0.90*	0.92	0.89**	0.85***	0.98	0.92	0.90	0.92	0.94	0.85	0.83*	0.70**	0.68**	0.68**	0.69**	0.66**
MF-DFM Median	0.91*	0.92**	0.89*	0.92	0.89**	0.85***	0.99	0.92	0.90	0.92	0.94	0.85	0.83*	0.71**	0.70**	0.70**	0.70**	0.67**
MF-DFM DMS	0.94*	1.07	1.04	0.98	1.04	1.02	0.97	1.06	0.96	1.12	1.14	0.91	1.10	0.73*	0.69*	0.64**	0.66*	0.67**
MF-DFM DMA	0.92*	0.98	0.93	0.91	0.98	0.95	0.98	1.02	1.01	1.06	1.10	0.91	0.99	0.67*	0.68*	0.63**	0.64**	0.67**
MF-DFM BMA	0.93	0.97	0.93	0.91*	0.99	0.92	1.02	0.98	1.07	1.02	1.01	0.84*	1.06	0.66*	0.61**	0.56**	0.56**	0.60***
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.97	0.98	0.97	0.97	0.88*	0.83*	0.78	0.75*	0.71*	0.76*	0.76*	0.72*	0.68*	0.60*	0.59*	0.58*	0.58*	0.56*
TVP-MF-DFM Median	0.97	0.98	0.97	0.97	0.87*	0.83*	0.77	0.75*	0.71	0.76*	0.75*	0.71*	0.67*	0.61*	0.60*	0.59*	0.59*	0.56*
TVP-MF-DFM DMS	0.99	1.02	0.98	0.96	0.93*	0.95	0.81	0.74	0.77	0.84	0.80	0.75*	0.89**	0.67	0.66	0.47	0.50	0.51
TVP-MF-DFM DMA	0.95	0.99	0.97	0.96*	0.89**	0.84*	0.80	0.78	0.77	0.83	0.77	0.73*	0.79*	0.61*	0.59*	0.50*	0.50*	0.53*
TVP-MF-DFM BMA	0.96	0.96	0.95	0.95**	0.86*	0.84**	0.75	0.73	0.72	0.77	0.74*	0.70*	0.74*	0.57*	0.60*	0.44*	0.45*	0.48*
MF-DFM Mean	0.93**	0.94**	0.92*	0.95*	0.90**	0.88**	0.84	0.80	0.74	0.76	0.77	0.72*	0.68*	0.58*	0.57*	0.56*	0.56*	0.55*
MF-DFM Median	0.93**	0.94**	0.92*	0.95*	0.89**	0.87**	0.84	0.81	0.73	0.76	0.78	0.72*	0.68*	0.59*	0.58*	0.58*	0.58*	0.56*
MF-DFM DMS	0.98	0.99	0.99	0.99	0.98	0.97	0.89	0.89	0.73	0.89	0.86	0.71	1.04	0.51	0.47	0.44*	0.44*	0.51*
MF-DFM DMA	0.98	0.96	0.95	0.97	0.98	0.97	0.88	0.87	0.77	0.88	0.87	0.74	0.91	0.47*	0.47*	0.43*	0.43*	0.52*
MF-DFM BMA	1.01	0.95	0.95	0.97	0.98	0.91*	0.91	0.82	0.86	0.8	0.79	0.72	0.92	0.47*	0.44*	0.41*	0.41*	0.48*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.8: Relative forecast performance, 1 Factors, $\gamma = 0.9$

h=	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	-1	-3	-5
Relative MAD																		
TVP-MF-DFM Mean	0.94**	0.94	0.93**	0.93	0.87***	0.83***	0.83**	0.83**	0.82**	0.81**	0.82*	0.77**	0.78**	0.70**	0.70**	0.69**	0.69**	0.68**
TVP-MF-DFM Median	0.94**	0.94	0.93*	0.93	0.87***	0.83***	0.84**	0.84**	0.82**	0.82**	0.82*	0.78**	0.79*	0.71**	0.70**	0.70**	0.70**	0.69**
TVP-MF-DFM DMS	0.96***	0.97	0.94***	0.95**	0.98	0.91**	0.88*	0.86**	0.93	0.95	0.92	0.84	0.88	0.80*	0.81	0.78	0.76	0.75*
TVP-MF-DFM DMA	0.96***	0.98	0.96**	0.97	0.92**	0.90**	0.88*	0.86*	0.89	0.87	0.88	0.83*	0.88	0.76*	0.76*	0.74*	0.74*	0.73**
TVP-MF-DFM BMA	0.97**	0.99	0.98	0.99	0.92**	0.90**	0.90	0.89*	0.88*	0.87	0.85*	0.81*	0.84*	0.77*	0.76*	0.72*	0.71**	0.74*
MF-DFM Mean	0.90**	0.91**	0.89*	0.92	0.85**	0.83***	1.01	0.92	1.00	0.94	1.00	0.91	0.94	0.78*	0.79*	0.79*	0.79*	0.76*
MF-DFM Median	0.90**	0.91**	0.89*	0.92*	0.84**	0.83***	1.01	0.93	0.98	0.93	0.99	0.91	0.94	0.80*	0.80	0.80*	0.80*	0.77*
MF-DFM DMS	0.91**	1.02	1.02	0.99	1.00	1.03	1.17	1.09	1.08	1.07	1.24	0.88	1.14	0.81*	0.77	0.69**	0.67**	0.67**
MF-DFM DMA	0.87***	0.94	0.92	0.93	0.97	1.02	1.16	1.05	1.09	1.06	1.02	0.89	0.93	0.79*	0.74*	0.68**	0.68**	0.66**
MF-DFM BMA	0.92	0.96	0.96	0.93	1.01	1.07	1.15	1.03	1.18	1.09	1.01	0.91	0.95	0.79*	0.70*	0.70**	0.70**	0.68**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.95**	0.95*	0.94**	0.95*	0.90**	0.86**	0.81*	0.77*	0.75*	0.72	0.77*	0.70*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM Median	0.95**	0.95*	0.94**	0.95*	0.89**	0.85**	0.81*	0.76*	0.74	0.71	0.76*	0.69*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM DMS	0.96*	0.96*	0.95*	0.95*	0.96	0.90*	0.77	0.76	0.86	0.82	0.80	0.71	0.81*	0.66	0.65	0.65	0.60	0.62*
TVP-MF-DFM DMA	0.96*	0.96	0.95*	0.95*	0.90*	0.88*	0.78	0.75	0.76	0.74	0.77	0.71	0.78	0.65*	0.66*	0.61*	0.61*	0.63*
TVP-MF-DFM BMA	0.96	0.97	0.96	0.96	0.91*	0.89*	0.80	0.76	0.77	0.74	0.76	0.69*	0.76*	0.64*	0.64*	0.59*	0.59*	0.62*
MF-DFM Mean	0.92**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.78	0.77	0.85	0.78	0.81	0.69*	0.69*	0.67*	0.67*	0.66*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.77	0.77	0.84	0.78	0.81	0.70*	0.70*	0.68*	0.68*	0.66*
MF-DFM DMS	0.95**	1.00	0.96	0.96	0.84	0.97	1.11	0.89	0.95	0.87	1.00	0.77	0.95	0.71*	0.57	0.57*	0.56*	0.58*
MF-DFM DMA	0.94**	0.98	0.94	0.95	0.84	0.96	1.11	0.88	0.97	0.86	0.84	0.78	0.81	0.68*	0.57*	0.56*	0.56*	0.58*
MF-DFM BMA	0.97	1.00	1.00	0.95	0.84	0.98	1.12	0.83	1.00	0.88	0.82	0.77	0.84	0.69*	0.54*	0.57*	0.57*	0.59*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.9: Relative forecast performance, 1 Factors, $\gamma = 0.8$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	0.94**	0.94	0.93**	0.93	0.87***	0.83***	0.83**	0.83**	0.82**	0.81**	0.82*	0.77**	0.78**	0.70**	0.70**	0.69**	0.69**	0.68**
TVP-MF-DFM Median	0.94**	0.94	0.93*	0.93	0.87***	0.83***	0.84**	0.84**	0.82**	0.82**	0.82*	0.78**	0.79*	0.71**	0.70**	0.70**	0.70**	0.69**
TVP-MF-DFM DMS	0.93***	0.94***	0.92***	0.95*	0.93**	0.90***	0.91	0.89*	0.93	0.95	0.97	0.88	0.99	0.84	0.84	0.76	0.78	0.75*
TVP-MF-DFM DMA	0.95***	0.96*	0.95**	0.96	0.90***	0.89***	0.89*	0.85**	0.88	0.86	0.89	0.83*	0.92	0.78*	0.78*	0.74*	0.75*	0.73*
TVP-MF-DFM BMA	0.97**	0.99	0.98	0.99	0.92**	0.90**	0.90	0.89*	0.88*	0.87	0.85*	0.81*	0.84*	0.77*	0.76*	0.72*	0.71**	0.74*
MF-DFM Mean	0.90**	0.91**	0.89*	0.92	0.85**	0.83***	1.01	0.92	1.00	0.94	1.00	0.91	0.94	0.78*	0.79*	0.79*	0.79*	0.76*
MF-DFM Median	0.90**	0.91**	0.89*	0.92*	0.84**	0.83***	1.01	0.93	0.98	0.93	0.99	0.91	0.94	0.80*	0.80	0.80*	0.80*	0.77*
MF-DFM DMS	0.91**	1.02	1.02	1.00	1.01	1.07	1.19	1.09	1.14	1.06	1.25	0.87	1.19	0.76**	0.80	0.68**	0.67**	0.66**
MF-DFM DMA	0.86***	0.94	0.91	0.93	0.98	1.03	1.17	1.07	1.09	1.06	1.03	0.87	0.95	0.77*	0.75*	0.67**	0.67**	0.67**
MF-DFM BMA	0.92	0.96	0.96	0.93	1.01	1.07	1.15	1.03	1.18	1.09	1.01	0.91	0.95	0.79*	0.70*	0.70**	0.70**	0.68**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.95**	0.95*	0.94**	0.95*	0.90**	0.86**	0.81*	0.77*	0.75*	0.72	0.77*	0.70*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM Median	0.95**	0.95*	0.94**	0.95*	0.89**	0.85**	0.81*	0.76*	0.74	0.71	0.76*	0.69*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM DMS	0.98***	0.98**	0.98***	0.96*	0.95*	0.92*	0.77	0.77	0.78	0.81	0.83	0.76	0.86	0.70	0.70	0.61	0.60	0.61*
TVP-MF-DFM DMA	0.96*	0.96*	0.95*	0.95*	0.90*	0.87*	0.79	0.75	0.76	0.74	0.78	0.72*	0.82	0.66*	0.67*	0.62*	0.63*	0.64*
TVP-MF-DFM BMA	0.96	0.97	0.96	0.96	0.91*	0.89*	0.80	0.76	0.77	0.74	0.76	0.69*	0.76*	0.64*	0.64*	0.59*	0.59*	0.62*
MF-DFM Mean	0.92**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.78	0.77	0.85	0.78	0.81	0.69*	0.69*	0.67*	0.67*	0.66*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.77	0.77	0.84	0.78	0.81	0.70*	0.70*	0.68*	0.68*	0.66*
MF-DFM DMS	0.96**	1.00	0.96	0.97	0.85	0.98	1.11	0.88	0.98	0.85	1.01	0.75*	1.05	0.70*	0.60	0.58*	0.57*	0.58*
MF-DFM DMA	0.94***	0.97	0.93	0.95	0.84	0.97	1.09	0.88	0.98	0.85	0.85	0.76*	0.85	0.65*	0.58	0.57*	0.58*	0.59*
MF-DFM BMA	0.97	1.00	1.00	0.95	0.84	0.98	1.12	0.83	1.00	0.88	0.82	0.77	0.84	0.69*	0.54*	0.57*	0.57*	0.59*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.10: Relative forecast performance, 1 Factors, $\gamma = 0.7$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	0.94**	0.94	0.93**	0.93	0.87***	0.83***	0.83**	0.83**	0.82**	0.81**	0.82*	0.77**	0.78**	0.70**	0.70**	0.69**	0.69**	0.68**
TVP-MF-DFM Median	0.94**	0.94	0.93*	0.93	0.87***	0.83***	0.84**	0.84**	0.82**	0.82**	0.82*	0.78**	0.79*	0.71**	0.70**	0.70**	0.70**	0.69**
TVP-MF-DFM DMS	0.95***	0.95**	0.94***	0.97	0.95**	0.90***	0.92	0.88*	0.91	0.92	0.93	0.90	0.97	0.85	0.86	0.77	0.79	0.75*
TVP-MF-DFM DMA	0.95***	0.96*	0.95***	0.96	0.89***	0.88***	0.88*	0.85**	0.89	0.86*	0.88	0.82*	0.90	0.77*	0.77*	0.74*	0.74*	0.74*
TVP-MF-DFM BMA	0.97**	0.99	0.98	0.99	0.92**	0.90**	0.90	0.89*	0.88*	0.87	0.85*	0.81*	0.84*	0.77*	0.76*	0.72*	0.71**	0.74*
MF-DFM Mean	0.90**	0.91**	0.89*	0.92	0.85**	0.83***	1.01	0.92	1.00	0.94	1.00	0.91	0.94	0.78*	0.79*	0.79*	0.79*	0.76*
MF-DFM Median	0.90**	0.91**	0.89*	0.92*	0.84**	0.83***	1.01	0.93	0.98	0.93	0.99	0.91	0.94	0.80*	0.80	0.80*	0.80*	0.77*
MF-DFM DMS	0.96	1.02	1.01	1.00	1.01	1.07	1.14	1.12	1.16	1.10	1.25	0.90	1.19	0.77*	0.78	0.70**	0.69**	0.67**
MF-DFM DMA	0.90*	0.94	0.91	0.93	0.99	1.02	1.16	1.10	1.12	1.08	1.03	0.90	0.97	0.77*	0.77	0.70**	0.70**	0.70**
MF-DFM BMA	0.92	0.96	0.96	0.93	1.01	1.07	1.15	1.03	1.18	1.09	1.01	0.91	0.95	0.79*	0.70*	0.70**	0.70**	0.68**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.95**	0.95*	0.94**	0.95*	0.90**	0.86**	0.81*	0.77*	0.75*	0.72	0.77*	0.70*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM Median	0.95**	0.95*	0.94**	0.95*	0.89**	0.85**	0.81*	0.76*	0.74	0.71	0.76*	0.69*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM DMS	0.99**	0.98*	0.98**	0.99	0.98**	0.92*	0.80	0.75	0.75	0.74	0.78	0.76	0.84	0.71	0.71	0.62	0.62	0.61*
TVP-MF-DFM DMA	0.96*	0.96*	0.95*	0.95*	0.90*	0.87*	0.79	0.76	0.77	0.73	0.78	0.72*	0.82	0.67*	0.68*	0.63*	0.64*	0.64*
TVP-MF-DFM BMA	0.96	0.97	0.96	0.96	0.91*	0.89*	0.80	0.76	0.77	0.74	0.76	0.69*	0.76*	0.64*	0.64*	0.59*	0.59*	0.62*
MF-DFM Mean	0.92**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.78	0.77	0.85	0.78	0.81	0.69*	0.69*	0.67*	0.67*	0.66*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.77	0.77	0.84	0.78	0.81	0.70*	0.70*	0.68*	0.68*	0.66*
MF-DFM DMS	0.97	1.00	0.95	0.97	0.85	0.98	1.07	0.90	1.00	0.87	1.01	0.76	1.05	0.69*	0.59	0.58*	0.58*	0.59*
MF-DFM DMA	0.95**	0.98	0.92	0.95	0.84	0.97	1.08	0.89	0.99	0.86	0.84	0.76	0.92	0.68*	0.60	0.59*	0.59*	0.61*
MF-DFM BMA	0.97	1.00	1.00	0.95	0.84	0.98	1.12	0.83	1.00	0.88	0.82	0.77	0.84	0.69*	0.54*	0.57*	0.57*	0.59*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.

Table A.11: Relative forecast performance, 1 Factors, $\gamma = 0.6$

h=	**29**	**27**	**25**	**23**	**21**	**19**	**17**	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**
Relative MAD																		
TVP-MF-DFM Mean	0.94**	0.94	0.93**	0.93	0.87***	0.83***	0.83**	0.83**	0.82**	0.81**	0.82*	0.77**	0.78**	0.70**	0.70**	0.69**	0.69**	0.68**
TVP-MF-DFM Median	0.94**	0.94	0.93*	0.93	0.87***	0.83***	0.84**	0.84**	0.82**	0.82**	0.82*	0.78**	0.79*	0.71**	0.70**	0.70**	0.70**	0.69**
TVP-MF-DFM DMS	0.95***	0.97	0.95**	1.03	0.93**	0.91**	0.93	0.91	0.96	0.92	0.92	0.89	0.92	0.84*	0.84	0.74*	0.76*	0.75*
TVP-MF-DFM DMA	0.95***	0.96*	0.95***	0.95*	0.88***	0.87***	0.89*	0.85**	0.88*	0.85*	0.88	0.81*	0.89	0.77*	0.77*	0.74**	0.74*	0.75*
TVP-MF-DFM BMA	0.97**	0.99	0.98	0.99	0.92**	0.90**	0.90	0.89*	0.88*	0.87	0.85*	0.81*	0.84*	0.77*	0.76*	0.72*	0.71**	0.74*
MF-DFM Mean	0.90**	0.91**	0.89*	0.92	0.85**	0.83***	1.01	0.92	1.00	0.94	1.00	0.91	0.94	0.78*	0.79*	0.79*	0.79*	0.76*
MF-DFM Median	0.90**	0.91**	0.89*	0.92*	0.84**	0.83***	1.01	0.93	0.98	0.93	0.99	0.91	0.94	0.80*	0.80	0.80*	0.80*	0.77*
MF-DFM DMS	0.96	1.02	1.01	0.99	1.02	1.09	1.12	1.13	1.17	1.09	1.23	0.92	1.17	0.75*	0.81	0.71**	0.72*	0.72**
MF-DFM DMA	0.90*	0.94	0.91	0.92	0.99	1.01	1.14	1.11	1.14	1.11	1.01	0.90	0.95	0.78*	0.80	0.74*	0.74*	0.73**
MF-DFM BMA	0.92	0.96	0.96	0.93	1.01	1.07	1.15	1.03	1.18	1.09	1.01	0.91	0.95	0.79*	0.70*	0.70**	0.70**	0.68**
F-MIDAS (Mean)							0.99	0.97	1.02	0.82*	0.97	0.72**	0.88	0.71**	0.76**	0.65***	0.73**	0.68**
MF-VAR	0.97	0.99	0.92	0.94	0.85**	0.86**	0.93	0.89	0.83*	0.83*	0.89	0.85*	0.87	0.81	0.78*	0.72**	0.73**	0.73**
Benchmark (abs.)	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Relative RMSE																		
TVP-MF-DFM Mean	0.95**	0.95*	0.94**	0.95*	0.90**	0.86**	0.81*	0.77*	0.75*	0.72	0.77*	0.70*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM Median	0.95**	0.95*	0.94**	0.95*	0.89**	0.85**	0.81*	0.76*	0.74	0.71	0.76*	0.69*	0.72*	0.64*	0.65*	0.62*	0.63*	0.60*
TVP-MF-DFM DMS	0.99**	0.99	0.98**	1.00	0.92	0.92*	0.80	0.75	0.80	0.77	0.78	0.75	0.84	0.70	0.71	0.61*	0.62*	0.61*
TVP-MF-DFM DMA	0.96**	0.96*	0.95*	0.95**	0.90*	0.87*	0.80	0.76	0.76	0.73	0.78	0.72*	0.82	0.67*	0.68*	0.64*	0.65*	0.65*
TVP-MF-DFM BMA	0.96	0.97	0.96	0.96	0.91*	0.89*	0.80	0.76	0.77	0.74	0.76	0.69*	0.76*	0.64*	0.64*	0.59*	0.59*	0.62*
MF-DFM Mean	0.92**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.78	0.77	0.85	0.78	0.81	0.69*	0.69*	0.67*	0.67*	0.66*
MF-DFM Median	0.93**	0.94**	0.92**	0.95**	0.85*	0.85**	0.81	0.80	0.77	0.77	0.84	0.78	0.81	0.70*	0.70*	0.68*	0.68*	0.66*
MF-DFM DMS	0.97	1.00	0.95	0.96	0.85	0.98	1.06	0.91	1.01	0.87	0.99	0.77	1.05	0.68*	0.61	0.59*	0.59*	0.63*
MF-DFM DMA	0.95**	0.98	0.93	0.94	0.84	0.97	1.06	0.90	0.99	0.87	0.83	0.76	0.92	0.69*	0.65	0.61*	0.61*	0.63*
MF-DFM BMA	0.97	1.00	1.00	0.95	0.84	0.98	1.12	0.83	1.00	0.88	0.82	0.77	0.84	0.69*	0.54*	0.57*	0.57*	0.59*
F-MIDAS (Mean)							0.91	0.84	0.88	0.74*	0.81	0.64*	0.73	0.58*	0.64*	0.58*	0.58*	0.60*
MF-VAR	0.96	0.97	0.93*	0.94	0.84*	0.85*	0.80	0.79*	0.78*	0.74*	0.84*	0.82*	0.75*	0.70*	0.71*	0.68*	0.69*	0.68*
Benchmark (abs.)	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Notes: See notes in Table A.1.